Chapter 8

Prerequisite Skills (p. 486)

1. 13^8: exponent = 8, base = 13
2. An expression that represents repeated multiplication of the same factor is called a *power*.
3. $10^2 = 10 \cdot 10 = 100$
4. $3^3 = 3 \cdot 3 \cdot 3 = 27$
5. $\left(\frac{5}{6}\right)^2 = \frac{5^2}{6^2} = \frac{25}{36}$
6. $\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$
7. 6.01, 6.12, 6.2
8. 0.0098, 0.073, 0.101
9. 4% = 04% = 0.04
10. 0.5% = 005% = 0.005
11. 13.8% = 13.8% = 0.138
12. 145% = 145% = 1.45
13. Let x be the input, or independent variable, and let y be the output, or dependent variable. Notice that each output is 2 more than the corresponding input. So, a rule for the function is $y = x + 2$.

Lesson 8.1

Investigating Algebra Activity 8.1 (p. 488)

Explore 1

Expression	Expression as repeated multiplication
$7^4 \cdot 7^5$	$(7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7 \cdot 7 \cdot 7)$
$(-4)^2 \cdot (-4)^3$	$[(-4) \cdot (-4)] \cdot [(-4) \cdot (-4) \cdot (-4)]$
$x^1 \cdot x^5$	$(x) \cdot (x \cdot x \cdot x \cdot x \cdot x)$

Expression	Number of factors	Simplified expression
$7^4 \cdot 7^5$	9	7^9
$(-4)^2 \cdot (-4)^3$	5	$(-4)^5$
$x^1 \cdot x^5$	6	x^6

Sample answer: The exponent of the simplified expression is equal to the sum of the exponents in the first column.

Explore 2

Expression	Expanded Expression
$(5^3)^2$	$(5^3) \cdot (5^3)$
$[(-6)^2]^4$	$(-6)^2 \cdot (-6)^2 \cdot (-6)^2 \cdot (-6)^2$
$(a^3)^3$	$(a^3) \cdot (a^3) \cdot (a^3)$

Expression	Expression as repeated multiplication
$(5^3)^2$	$(5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$
$[(-6)^2]^4$	$(-6)(-6)(-6)(-6)(-6)(-6)(-6)(-6)$
$(a^3)^3$	$(a \cdot a \cdot a) \cdot (a \cdot a \cdot a) \cdot (a \cdot a \cdot a)$

Expression	Number of factors	Simplified Expression
$(5^3)^2$	6	5^6
$[(-6)^2]^4$	8	$(-6)^8$
$(a^3)^3$	9	a^9

Sample answer: The exponent of the simplied expression is equal to the product of the exponents in the first column.

Draw Conclusions

1. $5^2 \cdot 5^3 = 5^{2+3} = 5^5$
2. $(-6)^1 \cdot (-6)^4 = (-6)^{1+4} = (-6)^5$
3. $m^6 \cdot m^4 = m^{6+4} = m^{10}$
4. $10^{3(3)} = 10^{3 \cdot 3} = 10^9$
5. $[(-2)^3]^4 = (-2)^{3 \cdot 4} = (-2)^{12}$
6. $c^{2(6)} = c^{2 \cdot 6} = c^{12}$
7. If a is a real number and m and n are positive integers, then $a^m \cdot a^n = a^{m+n}$.
8. If a is a real number and m and n are positive integers, then $(a^m)^n = a^{mn}$.

8.1 Guided Practice (pp. 489–491)

1. $3^2 \cdot 3^7 = 3^{2+7} = 3^9$
2. $5 \cdot 5^9 = 5^{1+9} = 5^{10}$
3. $(-7)^2(-7) = (-7)^{2+1} = (-7)^3$
4. $x^2 \cdot x^6 \cdot x = x^{2+6+1} = x^9$
5. $(4^2)^7 = 4^{2 \cdot 7} = 4^{14}$
6. $[(-2)^4]^5 = (-2)^{4 \cdot 5} = (-2)^{20}$
7. $(n^3)^6 = n^{3 \cdot 6} = n^{18}$
8. $[(m+1)^5]^4 = (m+1)^{5 \cdot 4} = (m+1)^{20}$
9. $(42 \cdot 12)^2 = 42^2 \cdot 12^2$
10. $(-3n)^2 = (-3 \cdot n)^2 = (-3)^2 \cdot n^2 = 9n^2$
11. $(9m^3n)^4 = (9 \cdot m^3 \cdot n)^4 = 9^4 \cdot m^{3 \cdot 4} \cdot n^4 = 6561m^{12}n^4$
12. $5 \cdot (5x^2)^4 = 5 \cdot (5 \cdot x^2)^4 = 5 \cdot 5^4 \cdot x^{2 \cdot 4} = 5^{1+4} \cdot x^8$
    $= 3125x^8$
13. $10^2 \cdot 10^4 = 10^{2+4} = 10^6$
    About 10^6, or 1,000,000, bees were studied in Idaho.

8.1 Exercises (pp. 492–494)

Skill Practice

1. The *order of magnitude* of the quantity 93,534,004 people is the power of 10 nearest the quantity, or 10^8 people.
2. When powers have the same base, their product is the base raised to the sum of the exponents.
3. $4^2 \cdot 4^6 = 4^{2+6} = 4^8$
4. $8^5 \cdot 8^2 = 8^{5+2} = 8^7$
5. $3^3 \cdot 3 = 3^{3+1} = 3^4$
6. $9 \cdot 9^5 = 9^{1+5} = 9^6$
7. $(-7)^4(-7)^5 = (-7)^{4+5} = (-7)^9$
8. $(-6)^6(-6) = (-6)^{6+1} = (-6)^7$
9. $2^4 \cdot 2^9 \cdot 2 = 2^{4+9+1} = 2^{14}$
10. $(-3)^2(-3)^{11}(-3) = (-3)^{2+11+1} = (-3)^{14}$

11. $(3^5)^2 = 3^{5 \cdot 2} = 3^{10}$

12. $(7^4)^3 = 7^{4 \cdot 3} = 7^{12}$

13. $[(-5)^3]^4 = (-5)^{3 \cdot 4} = (-5)^{12}$

14. $[(-8)^9]^2 = (-8)^{9 \cdot 2} = (-8)^{18}$

15. $(15 \cdot 29)^3 = 15^3 \cdot 29^3$

16. $(17 \cdot 16)^4 = 17^4 \cdot 16^4$

17. $(132 \cdot 9)^6 = 132^6 \cdot 9^6$

18. $((-14) \cdot 22)^5 = (-14)^5 \cdot 22^5$

19. $x^4 \cdot x^2 = x^{4+2} = x^6$

20. $y^9 \cdot y = y^{9+1} = y^{10}$

21. $z^2 \cdot z \cdot z^3 = z^{2+1+3} = z^6$

22. $a^4 \cdot a^3 \cdot a^{10} = a^{4+3+10} = a^{17}$

23. $(x^5)^2 = x^{5 \cdot 2} = x^{10}$

24. $(y^4)^6 = y^{4 \cdot 6} = y^{24}$

25. $[(b-2)^2]^6 = (b-2)^{2 \cdot 6} = (b-2)^{12}$

26. $[(d+9)^7]^3 = (d+9)^{7 \cdot 3} = (d+9)^{21}$

27. $(-5x)^2 = (-5)^2 \cdot x^2 = 25x^2$

28. $-(5x)^2 = -(5^2 \cdot x^2) = -25x^2$

29. $(7xy)^2 = 7^2 \cdot x^2 \cdot y^2 = 49x^2y^2$

30. $(5pq)^3 = 5^3 \cdot p^3 \cdot q^3 = 125p^3q^3$

31.
$$\begin{aligned}(-10x^6)^2 \cdot x^2 &= (-10)^2 \cdot x^{6 \cdot 2} \cdot x^2\\ &= 100 \cdot x^{12} \cdot x^2\\ &= 100 \cdot x^{12+2}\\ &= 100x^{14}\end{aligned}$$

32.
$$\begin{aligned}(-8m^4)^2 \cdot m^3 &= (-8)^2 \cdot m^{4 \cdot 2} \cdot m^3\\ &= 64 \cdot m^8 \cdot m^3\\ &= 64m^{8+3}\\ &= 64m^{11}\end{aligned}$$

33.
$$\begin{aligned}6d^2 \cdot (2d^5)^4 &= 6d^2 \cdot 2^4 \cdot d^{5 \cdot 4}\\ &= 6d^2 \cdot 16 \cdot d^{20}\\ &= 6 \cdot 16 \cdot d^{2+20}\\ &= 96d^{22}\end{aligned}$$

34.
$$\begin{aligned}(-20x^3)^2(-x^7) &= (-20)^2 \cdot x^{3 \cdot 2} \cdot (-x^7)\\ &= 400 \cdot x^6 \cdot (-1) \cdot x^7\\ &= 400 \cdot (-1) \cdot x^{6+7}\\ &= -400x^{13}\end{aligned}$$

35.
$$\begin{aligned}-(2p^4)^3(-1.5p^7) &= -(2^3 \cdot p^{4 \cdot 3}) \cdot (-1.5 \cdot p^7)\\ &= -(8 \cdot p^{12}) \cdot (-1.5 \cdot p^7)\\ &= (-8) \cdot (-1.5) \cdot p^{12} \cdot p^7\\ &= 12p^{12+7}\\ &= 12p^{19}\end{aligned}$$

36.
$$\begin{aligned}\left(\tfrac{1}{2}y^5\right)^3(2y^2)^4 &= \left(\tfrac{1}{2}\right)^3 \cdot y^{5 \cdot 3} \cdot 2^4 \cdot y^{2 \cdot 4}\\ &= \tfrac{1}{8} \cdot y^{15} \cdot 16 \cdot y^8\\ &= 2y^{15+8}\\ &= 2y^{23}\end{aligned}$$

37.
$$\begin{aligned}(3x^5)^3(2x^7)^2 &= 3^3 \cdot x^{5 \cdot 3} \cdot 2^2 \cdot x^{7 \cdot 2}\\ &= 27 \cdot x^{15} \cdot 4 \cdot x^{14}\\ &= 108x^{15+14}\\ &= 108x^{29}\end{aligned}$$

38.
$$\begin{aligned}(-10n)^2(-4n^3)^3 &= (-10)^2 \cdot n^2 \cdot (-4)^3 \cdot n^{3 \cdot 3}\\ &= 100 \cdot n^2 \cdot (-64) \cdot n^9\\ &= -6400n^{2+9}\\ &= -6400n^{11}\end{aligned}$$

39. The exponents were multiplied instead of added;
$c \cdot c^4 \cdot c^5 = c^1 \cdot c^4 \cdot c^5 = c^{1+4+5} = c^{10}$

40. B; $(-9)(-9)^5 = (-9)^{1+5} = (-9)^6$

41. D;
$$\begin{aligned}(6x^5)^2 \cdot x^2 &= 6^2 \cdot x^{5 \cdot 2} \cdot x^2\\ &= 36 \cdot x^{10} \cdot x^2\\ &= 36 \cdot x^{10+2}\\ &= 36x^{12}\end{aligned}$$

42.
$$\begin{aligned}x^4 \cdot x^? &= x^5\\ 4 + ? &= 5\\ ? &= 1\end{aligned}$$

43.
$$\begin{aligned}(y^8)^? &= y^{16}\\ 8 \cdot ? &= 16\\ ? &= 2\end{aligned}$$

44.
$$\begin{aligned}(2z^?)^3 &= 8z^{15}\\ 2^3 \cdot z^{? \cdot 3} &= 8z^{15}\\ 8 \cdot z^{? \cdot 3} &= 8z^{15}\\ ? \cdot 3 &= 15\\ ? &= 5\end{aligned}$$

45.
$$\begin{aligned}(3a^3)^? \cdot 2a^3 &= 18a^9\\ 3^? \cdot a^{3 \cdot ?} \cdot 2 \cdot a^3 &= 18a^9\\ 3^? \cdot 2 \cdot a^{3 \cdot ?} \cdot a^3 &= 18a^9\\ a^{3 \cdot ? + 3} &= a^9\\ 3 \cdot ? + 3 &= 9\\ 3 \cdot ? &= 6\\ ? &= 2\end{aligned}$$

46. 10^7 people

47.
$$\begin{aligned}&(-3x^2y)^3(11x^3y^5)^2\\ &= (-3)^3 \cdot x^{2 \cdot 3} \cdot y^3 \cdot 11^2 \cdot x^{3 \cdot 2} \cdot y^{5 \cdot 2}\\ &= -27 \cdot x^6 \cdot y^3 \cdot 121 \cdot x^6 \cdot y^{10}\\ &= -3267 \cdot x^{6+6} \cdot y^{3+10}\\ &= -3267x^{12}y^{13}\end{aligned}$$

48.
$$\begin{aligned}&-(-xy^2z^3)^5(x^4yz)^2\\ &= -[(-1)^5 \cdot x^5 \cdot y^{2 \cdot 5} \cdot z^{3 \cdot 5}](x^{4 \cdot 2} \cdot y^2 \cdot z^2)\\ &= x^5 \cdot y^{10} \cdot z^{15} \cdot x^8 \cdot y^2 \cdot z^2\\ &= x^{5+8} \cdot y^{10+2} \cdot z^{15+2}\\ &= x^{13}y^{12}z^{17}\end{aligned}$$

49.
$$\begin{aligned}&(-2s)(-5r^3st)^3(-2r^4st^7)^2\\ &= -2s \cdot (-5)^3 \cdot r^{3 \cdot 3} \cdot s^3 \cdot t^3 \cdot (-2)^2 \cdot r^{4 \cdot 2} \cdot s^2 \cdot t^{7 \cdot 2}\\ &= -2s \cdot (-125) \cdot r^9 \cdot s^3 \cdot t^3 \cdot 4 \cdot r^8 \cdot s^2 \cdot t^{14}\\ &= (-2)(-125)(4) \cdot s^{1+3+2} \cdot r^{9+8} \cdot t^{3+14}\\ &= 1000r^{17}s^6t^{17}\end{aligned}$$

50. *Sample answer:* $3(2x^4)^2$, $12x \cdot x^7$, $12(x^4)^2$

51. $(a \cdot b)^m = (a \cdot b) \cdot (a \cdot b) \cdot \ldots \cdot (a \cdot b)$, with $a \cdot b$ as a factor m times. The associative and commutative properties of multiplication allow you to rewrite this expression as $a \cdot a \cdot \ldots \cdot a \cdot b \cdot b \cdot \ldots \cdot b$ with each factor appearing m times. This expression is equal to $a^m \cdot b^m$.

Problem Solving

52. $\frac{10^6 \text{ bubbles}}{1 \text{ cm}^3}\left(\frac{10^3 \text{ cm}^3}{1 \text{ quart}}\right) = 10^{6+3} = 10^9$ bubbles in 1 quart

53. $(10^{13})(10^{13}) = 10^{13+13} = 10^{26}$ m

54. $\frac{10^9 \text{ grains of sand}}{1 \text{ ft}^3}(10^7 \text{ ft}^3) = 10^{9+7} = 10^{16}$ grains of sand

55. a.

Gold (ounces)	10	100	1000	10,000	100,000
Number of atoms	10^{24}	10^{25}	10^{26}	10^{27}	10^{28}

b. $\frac{10^{23} \text{ atoms}}{1 \text{ ounce}}(10^5 \text{ ounces}) = 10^{23+5} = 10^{28}$ atoms

56. a. $(10^2)(10) = 10^{2+1} = 10^3$

b. $(10^2 \text{ nanometers})(10^3) = 10^{2+3} = 10^5$ nanometers

57. $(10^9 \text{ meters})^3 = 10^{9 \cdot 3} = 10^{27}\, m^3$

58. a. $(10^3)(10) = 10^{3+1} = 10^4$ ft

b. $V = (1)(10^3)^2(10^4) = 10^6(10^4) = 10^{6+4} = 10^{10}$ ft^3

c. The volume will be 10^2 greater because in the formula for volume, the radius is squared. So,

$$V = (1)(10^3 \cdot 10)^2(10^4)$$
$$= (10^4)^2(10^4)$$
$$= (10^8)(10^4)$$
$$= 10^{12} \text{ ft}^3$$

59. $2^{13}; 2^{10}; 2^{13+10} = 2^{23}$

Mixed Review

60. $\left(\frac{1}{2}\right)\left(-\frac{4}{5}\right) = \frac{1(-4)}{2(5)} = \frac{-4}{10} = -\frac{2}{5}$

61. $\left(-\frac{2}{3}\right)\left(\frac{7}{4}\right) = \frac{(-2)(7)}{3(4)} = \frac{-14}{12} = \frac{-7}{6}$

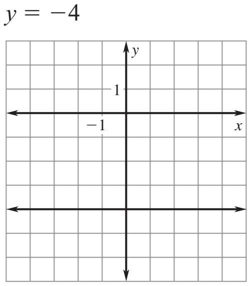
62. $\left(-\frac{6}{5}\right)\left(-\frac{3}{8}\right) = \frac{(-6)(-3)}{5(8)} = \frac{18}{40} = \frac{9}{20}$

63. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

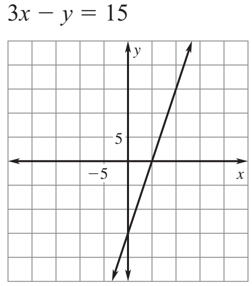
64. $(-2.2)^2 = (-2.2) \cdot (-2.2) = 4.84$

65. $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$

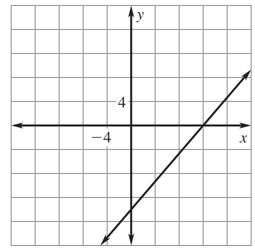
66. $y = -4$



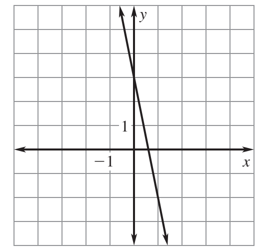
67. $3x - y = 15$



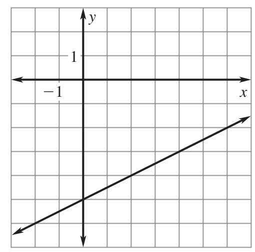
68. $7x - 6y = 84$



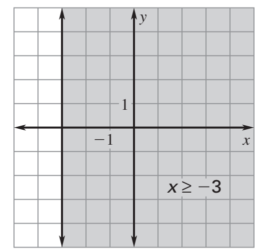
69. $y = -5x + 3$



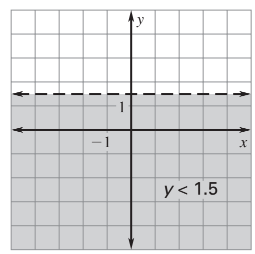
70. $y = \frac{1}{2}x - 5$



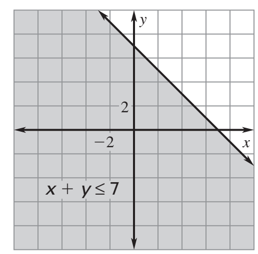
71. $x \ge -3$



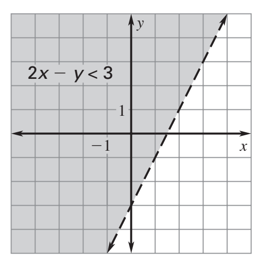
72. $y < 1.5$



73. $x + y \le 7$



74. $2x - y < 3$



Lesson 8.2

8.2 Guided Practice (pp. 495–498)

1. $\frac{6^{11}}{6^5} = 6^{11-5} = 6^6$

2. $\frac{(-4)^9}{(-4)^2} = (-4)^{9-2} = (-4)^7$

3. $\frac{9^4 \cdot 9^3}{9^2} = \frac{9^7}{9^2} = 9^{7-2} = 9^5$

4. $\frac{1}{y^5} \cdot y^8 = \frac{y^8}{y^5} = y^{8-5} = y^3$

5. $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$

6. $\left(-\frac{5}{y}\right)^3 = \left(\frac{-5}{y}\right)^3 = \frac{(-5)^3}{y^3} = \frac{-125}{y^3}$

7. $\left(\frac{x^2}{4y}\right)^2 = \frac{(x^2)^2}{(4y)^2} = \frac{x^4}{4^2y^2} = \frac{x^4}{16y^2}$

8. $\left(\frac{2s}{3t}\right)^3 \cdot \left(\frac{t^5}{16}\right) = \frac{(2s)^3}{(3t)^3} \cdot \frac{t^5}{16}$

$= \frac{2^3s^3}{3^3t^3} \cdot \frac{t^5}{16}$

$= \frac{8s^3}{27t^3} \cdot \frac{t^5}{16}$

$= \frac{8s^3t^5}{432t^3} = \frac{s^3t^2}{54}$

9.

Step	Number of new branches	Length of new branch
1	$3 = 3^1$	$\frac{1}{2} = \left(\frac{1}{2}\right)^1$
2	$9 = 3^2$	$\frac{1}{2}\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2$
3	$27 = 3^3$	$\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3$
4	$81 = 3^4$	$\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4$

The length of the new branch added at Step 9 is

$\left(\frac{1}{2}\right)^9 = \frac{1^9}{2^9} = \frac{1}{512}$.

10. $\frac{\text{Luminosity of Canopus (watts)}}{\text{Luminosity of Sirius(watts)}} = \frac{10^{30}}{10^{28}} = 10^{30-28} = 10^2$

Canopus is about 10^2 times as luminous as Sirius.

8.2 Exercises (pp. 498–501)

Skill Practice

1. In the power 4^3, 4 is the *base* and 3 is the *exponent*.

2. When powers have the same base, their quotient is the base raised to the difference of the exponents.

3. $\frac{5^6}{5^2} = 5^{6-2} = 5^4$

4. $\frac{2^{11}}{2^6} = 2^{11-6} = 2^5$

5. $\frac{3^9}{3^5} = 3^{9-5} = 3^4$

6. $\frac{(-6)^8}{(-6)^5} = (-6)^{8-5} = (-6)^3$

7. $\frac{(-4)^7}{(-4)^4} = (-4)^{7-4} = (-4)^3$

8. $\frac{(-12)^9}{(-12)^3} = (-12)^{9-3} = (-12)^6$

9. $\frac{10^5 \cdot 10^5}{10^4} = \frac{10^{5+5}}{10^4} = \frac{10^{10}}{10^4} = 10^{10-4} = 10^6$

10. $\frac{6^7 \cdot 6^4}{6^6} = \frac{6^{7+4}}{6^6} = \frac{6^{11}}{6^6} = 6^{11-6} = 6^5$

11. $\left(\frac{1}{3}\right)^5 = \frac{1^5}{3^5} = \frac{1}{3^5}$

12. $\left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4}$

13. $\left(-\frac{5}{4}\right)^4 = \frac{(-5)^4}{4^4} = \frac{5^4}{4^4}$

14. $\left(-\frac{2}{5}\right)^5 = \frac{(-2)^5}{5^5} = \frac{-2^5}{5^5}$

15. $7^9 \cdot \frac{1}{7^2} = \frac{7^9}{7^2} = 7^{9-2} = 7^7$

16. $\frac{1}{9^5} \cdot 9^{11} = \frac{9^{11}}{9^5} = 9^{11-5} = 9^6$

17. $\left(\frac{1}{3}\right)^4 \cdot 3^{12} = \frac{1^4}{3^4} \cdot 3^{12} = \frac{1}{3^4} \cdot 3^{12} = \frac{3^{12}}{3^4} = 3^{12-4} = 3^8$

18. $4^9 \cdot \left(-\frac{1}{4}\right)^5 = 4^9 \cdot \frac{(-1)^5}{4^5}$

$= 4^9 \cdot (-1) \cdot \frac{1}{4^5}$

$= -\frac{4^9}{4^5}$

$= -4^{9-5}$

$= -4^4$

19. C; $\left(\frac{16^6}{16^3}\right)^2 = (16^{6-3})^2 = (16^3)^2 = 16^{3 \cdot 2} = 16^6$

20. The quotient of powers property was used incorrectly. The exponents should be subtracted, not added;

$\frac{9^5 \cdot 9^3}{9^4} = \frac{9^8}{9^4} = 9^4$

21. $\frac{1}{y^8} \cdot y^{15} = \frac{y^{15}}{y^8} = y^{15-8} = y^7$

22. $z^8 \cdot \frac{1}{z^7} = \frac{z^8}{z^7} = z^{8-7} = z$

23. $\left(\frac{a}{y}\right)^9 = \frac{a^9}{y^9}$

24. $\left(\frac{j}{k}\right)^{11} = \frac{j^{11}}{k^{11}}$

25. $\left(\frac{p}{q}\right)^4 = \frac{p^4}{q^4}$

26. $\left(-\frac{1}{x}\right)^5 = \frac{(-1)^5}{x^5} = -\frac{1}{x^5}$

27. $\left(-\frac{4}{x}\right)^3 = \frac{(-4)^3}{x^3} = -\frac{64}{x^3}$

28. $\left(-\frac{a}{b}\right)^4 = (-1)^4 \cdot \left(\frac{a^4}{b^4}\right) = \frac{a^4}{b^4}$

29. $\left(\frac{4c}{d^2}\right)^3 = \frac{(4c)^3}{(d^2)^3} = \frac{4^3c^3}{d^6} = \frac{64c^3}{d^6}$

30. $\left(\frac{a^7}{2b}\right)^5 = \frac{(a^7)^5}{(2b)^5} = \frac{a^{35}}{2^5b^5} = \frac{a^{35}}{32b^5}$

31. $\left(\frac{x^2}{3y^3}\right)^2 = \frac{(x^2)^2}{(3y^3)^2} = \frac{x^4}{9y^6}$

32. $\left(\frac{3x^5}{7y^2}\right)^3 = \frac{(3x^5)^3}{(7y^2)^3} = \frac{3^3x^{15}}{7^3y^6} = \frac{27x^{15}}{343y^6}$

33. $\left(\frac{3x^3}{2y}\right)^2 \cdot \frac{1}{x^2} = \frac{3^2x^6}{2^2y^2} \cdot \frac{1}{x^2} = \frac{9x^6}{4y^2} \cdot \frac{1}{x^2} = \frac{9x^6}{4y^2x^2} = \frac{9x^{6-2}}{4y^2} = \frac{9x^4}{4y^2}$

34. $\left(\frac{2x^3}{y}\right)^3 \cdot \frac{1}{6x^3} = \frac{(2x^3)^3}{y^3} \cdot \frac{1}{6x^3}$

$= \frac{2^3x^9}{y^3} \cdot \frac{1}{6x^3}$

$= \frac{8x^9}{6y^3x^3}$

$= \frac{8x^{9-3}}{6y^3}$

$= \frac{4x^6}{3y^3}$

35. $\frac{3}{8m^5} \cdot \left(\frac{m^4}{n^2}\right)^3 = \frac{3}{8m^5} \cdot \frac{(m^4)^3}{(n^2)^3}$

$= \frac{3}{8m^5} \cdot \frac{m^{12}}{n^6}$

$= \frac{3m^{12}}{8m^5n^6}$

$= \frac{3m^{12-5}}{8n^6}$

$= \frac{3m^7}{8n^6}$

36. $\left(\frac{-5}{x}\right)^2 \cdot \left(\frac{2x^4}{y^3}\right)^2 = (-1)^2 \cdot \left(\frac{5^2}{x^2}\right) \cdot \frac{(2x^4)^2}{(y^3)^2}$

$= 1 \cdot \left(\frac{25}{x^2}\right) \cdot \frac{4x^8}{y^6}$

$= \frac{100x^8}{x^2y^6}$

$= \frac{100x^{8-2}}{y^6}$

$= \frac{100x^6}{y^6}$

37. D; $\left(\frac{7x^3}{2y^4}\right)^2 = \frac{(7x^3)^2}{(2y^4)^2} = \frac{49x^6}{4y^8}$

38. $\frac{(-8)^7}{(-8)^?} = (-8)^3$

$(-8)^{7-?} = (-8)^3$

$7 - ? = 3$

$? = 4$

39. $\frac{7^? \cdot 7^2}{7^4} = 7^6$

$\frac{7^{?+2}}{7^4} = 7^6$

$7^{?+2-4} = 7^6$

$? - 2 = 6$

$? = 8$

40. $\frac{1}{p^5} \cdot p^? = p^9$

$\frac{p^?}{p^5} = p^9$

$p^{?-5} = p^9$

$? - 5 = 9$

$? = 14$

41. $\left(\frac{2c^3}{d^2}\right)^? = \frac{16c^{12}}{d^8}$

$\frac{(2c^3)^?}{(d^2)^?} = \frac{16c^{12}}{d^8}$

$\frac{2^?c^{3?}}{d^{2?}} = \frac{16c^{12}}{d^8}$

$3? = 12$

$? = 4$

42. $\left(\frac{2f^2g^3}{3fg}\right)^4 = \frac{2^4(f^2)^4(g^3)^4}{3^4f^4g^4}$

$= \frac{16f^8g^{12}}{81f^4g^4}$

$= \frac{16f^{8-4}g^{12-4}}{81}$

$= \frac{16f^4g^8}{81}$

43. $\frac{2s^3t^3}{st^2} \cdot \frac{(3st)^3}{s^2t} = \frac{2s^3t^3}{st^2} \cdot \frac{3^3s^3t^3}{s^2t}$

$= \frac{2s^3t^3}{st^2} \cdot \frac{27s^3t^3}{s^2t}$

$= \frac{2s^{3-1}t^{3-2}}{1} \cdot \frac{27s^{3-2}t^{3-1}}{1}$

$= 2s^2t^1 \cdot 27st^2$

$= 54s^{2+1}t^{1+2} = 54s^3t^3$

44. $\left(\frac{2m^5n}{4m^2}\right)^2 \cdot \left(\frac{mn^4}{5n}\right)^2 = \frac{(2m^5n)^2}{(4m^2)^2} \cdot \frac{(mn^4)^2}{(5n)^2}$

$= \frac{4m^{10}n^2}{16m^4} \cdot \frac{m^2n^8}{25n^2}$

$= \frac{m^{10-4}n^2}{4} \cdot \frac{m^2n^{8-2}}{25}$

$= \frac{m^6n^6}{4} \cdot \frac{m^2n^6}{25} = \frac{m^{6+2}n^{2+6}}{100} = \frac{m^8n^8}{100}$

45. $\left(\frac{3x^3y}{x^2}\right)^3 \cdot \left(\frac{y^2x^4}{5y}\right)^2 = \frac{(3x^3y)^3}{(x^2)^3} \cdot \frac{(y^2x^4)^2}{(5y)^2}$

$= \frac{27x^9y^3}{x^6} \cdot \frac{y^4x^8}{25y^2}$

$= 27x^{9-6}y^3 \cdot \frac{y^{4-2}x^8}{25}$

$= 27x^3y^3 \cdot \frac{y^2x^8}{25}$

$= \frac{27x^{3+8}y^{3+2}}{25}$

$= \frac{27x^{11}y^5}{25}$

46. *Sample answer*: $\frac{14^9}{14^2}$; $\frac{14^{11}}{14^4}$; $\frac{14^5 \cdot 14^4}{14^2}$

47. Let $m < n$. Given

$\frac{a^m}{a^n} = \frac{a^m}{a^n}\left(\frac{\frac{1}{a^m}}{\frac{1}{a^m}}\right)$ Identity property of multiplication.

$= \frac{1}{\frac{a^n}{a^m}}$ Multiply fractions.

$= \frac{1}{a^{n-m}}$ Quotient of powers property.

48. $\frac{b^x}{b^y} = b^9$

$b^{x-y} = b^9$

$x - y = 9$ Equation 1

$\frac{b^x \cdot b^2}{b^{3y}} = b^{13}$

$\frac{b^{x+2}}{b^{3y}} = b^{13}$

$b^{x+2-3y} = b^{13}$

$x + 2 - 3y = 13$

$x - 3y = 11$ Equation 2

Solve the system $x - y = 9$

$x - 3y = 11$

$x = 9 + y$ Revised Equation 1

$(9 + y) - 3y = 11$ Substitute revised Eqn 1 into Eqn 2.

$9 - 2y = 11$

$-2y = 2$

$y = -1$

$x - (-1) = 9$ Substitute $y = -1$ into Eqn 1.

$x + 1 = 9$

$x = 8$

Problem Solving

49. a.

steps	1	2	3	4
new squares	4^1	4^2	4^3	4^4
side length	$\left(\frac{1}{2}\right)^1$	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{2}\right)^4$

b. $\frac{4^4}{4^2} = 4^{4-2} = 4^2 = 16$ times more squares

50. $\frac{10^{13}}{10^8} = 10^{13-8} = \$10^5$ per capital GDP

51. $10^{13}\text{ km} \cdot \frac{10^3\text{ m}}{1\text{ km}} = 10^{16}\text{ m}$

$\frac{10^{16}\text{ m}}{10^4\text{ m/sec}} = 10^{16-4} = 10^{12}\text{ sec}$

$10^{12}\text{ sec} \cdot \frac{1\text{ min}}{60\text{ sec}} \cdot \frac{1\text{ h}}{60\text{ min}} \cdot \frac{1\text{ day}}{24\text{ h}} \cdot \frac{1\text{ yr}}{365\text{ days}} \approx 31{,}710\text{ yr}$

52. $2.512^{5-2} = 2.512^3 \approx 15.85$ times less bright

53. $31^{7-4} = 31^3 = 29{,}791$ times greater

54. a. $\frac{2^{40}}{2^{10}} = 2^{40-10} = 2^{30}$ gigabytes

b. $\frac{2^{50}}{2^{20}} = 2^{50-20} = 2^{30}$ megabytes

c. Multiply the number of bytes by 8.

Mixed Review

55. $\frac{3}{4}k = 9$

$\frac{4}{3} \cdot \frac{3}{4}k = \frac{4}{3} \cdot 9$

$k = \frac{36}{3} = 12$

56. $\frac{2}{5}t = -4$

$\frac{5}{2} \cdot \frac{2}{5}t = \frac{5}{2} \cdot (-4)$

$t = \frac{-20}{2} = -10$

57. $-\frac{2}{3}v = 14$

$\left(-\frac{3}{2}\right)\left(-\frac{2}{3}v\right) = \left(-\frac{3}{2}\right)(14)$

$v = \frac{-42}{2} = -21$

58. $-\frac{5}{2}y = -35$

$\left(-\frac{2}{5}\right)\left(-\frac{5}{2}y\right) = \left(-\frac{2}{5}\right)(-35)$

$y = \frac{70}{5} = 14$

59. $-\frac{7}{5}z = \frac{14}{3}$

$\left(-\frac{5}{7}\right)\left(-\frac{7}{5}z\right) = \left(-\frac{5}{7}\right)\left(\frac{14}{3}\right)$

$z = -\frac{70}{21} = -\frac{10}{3}$

60. $-\frac{3}{2}z = -\frac{3}{4}$

$\left(-\frac{2}{3}\right)\left(-\frac{3}{2}z\right) = \left(-\frac{2}{3}\right)\left(-\frac{3}{4}\right)$

$z = \frac{6}{12} = \frac{1}{2}$

61. points: $(-2, 1), (0, -5)$

$m = \frac{-5 - 1}{0 - (-2)} = \frac{-6}{2} = -3$

$y - 1 = -3(x - (-2))$

$y - 1 = -3(x + 2)$

$y = -3x - 5$

62. points: $(0, 3), (-4, 1)$

$m = \frac{1 - 3}{-4 - 0} = \frac{-2}{-4} = \frac{1}{2}$

$y - 3 = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x + 3$

63. points: $(0, -3), (7, -3)$

$m = \frac{-3 - (-3)}{7 - 0} = \frac{0}{7} = 0$

$y - (-3) = 0(x - 0)$

$y = -3$

64. points: $(4, 3), (5, 6)$

$m = \frac{6 - 3}{5 - 4} = 3$

$y - 3 = 3(x - 4)$

$y = 3x - 9$

65. points: $(4, 1), (-2, 4)$

$m = \frac{4 - 1}{-2 - 4} = \frac{3}{-6} = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 4)$

$y = -\frac{1}{2}x + 3$

66. points: $(-1, -3), (-3, 1)$

$m = \frac{1 - (-3)}{-3 - (-1)} = \frac{4}{-2} = -2$

$y - (-3) = -2(x - (-1))$

$y + 3 = -2(x + 1)$

$y = -2x - 5$

Quiz 8.1–8.2 (p. 501)

1. $3^2 \cdot 3^6 = 3^{2+6} = 3^8$
2. $(5^4)^3 = 5^{4 \cdot 3} = 5^{12}$
3. $(32 \cdot 14)^7 = 32^7 \cdot 14^7$
4. $7^2 \cdot 7^6 \cdot 7 = 7^{2+6+1} = 7^9$
5. $(-4)(-4)^9 = (-4)^{1+9} = (-4)^{10}$
6. $\frac{7^{12}}{7^4} = 7^{12-4} = 7^8$
7. $\frac{(-9)^9}{(-9)^7} = (-9)^{9-7} = (-9)^2$
8. $\frac{3^7 \cdot 3^4}{3^6} = \frac{3^{7+4}}{3^6} = \frac{3^{11}}{3^6} = 3^{11-6} = 3^5$
9. $\left(\frac{5}{4}\right)^4 = \frac{5^4}{4^4}$
10. $x^2 \cdot x^5 = x^{2+5} = x^7$
11. $(3x^3)^2 = 3^2x^{3 \cdot 2} = 9x^6$
12. $-(7x)^2 = -7^2x^2 = -49x^2$
13. $(6x^5)^3 \cdot x = 6^3x^{5 \cdot 3} \cdot x = 216x^{15} \cdot x = 216x^{16}$
14. $(2x^5)^3(7x^7)^2 = 2^3x^{5 \cdot 3} \cdot 7^2x^{7 \cdot 2}$
$= 8x^{15} \cdot 49x^{14}$
$= 392x^{15+14}$
$= 392x^{29}$
15. $\frac{1}{x^9} \cdot x^{21} = \frac{x^{21}}{x^9} = x^{21-9} = x^{12}$
16. $\left(\frac{-4}{x}\right)^3 = \frac{(-4)^3}{x^3} = \frac{-64}{x^3}$
17. $\left(\frac{w}{v}\right)^6 = \frac{w^6}{v^6}$
18. $\left(\frac{x^3}{4}\right)^2 = \frac{(x^3)^2}{4^2} = \frac{x^6}{16}$
19. $\frac{10^{10} \text{ lb}}{10^6 \text{ acres}} = 10^{10-6} = 10^4$, about 10^4 pounds

Lesson 8.3

Investigating Algebra Activity 8.3 (p. 502)

Explore

Exponent, n	Value of 2^n
4	16
3	8
2	4
1	2

Exponent, n	Value of 3^n
4	81
3	27
2	9
1	3

Each time the exponent is decreased by 1, the value is divided by the base.

Exponent, n	Power, 2^n
3	8
2	4
1	2
0	1
−1	$\frac{1}{2}$
−2	$\frac{1}{4}$

Exponent, n	Power, 2^n
3	27
2	9
1	3
0	1
−1	$\frac{1}{3}$
−2	$\frac{1}{9}$

Draw Conclusions

1.

n	2^n	3^n
−3	$\frac{1}{8}$	$\frac{1}{27}$
−4	$\frac{1}{16}$	$\frac{1}{81}$
−5	$\frac{1}{32}$	$\frac{1}{243}$

2. $a^0 = 1$

3.

Power, 2^n	Exponent
8	2^3
4	2^2
2	2^1
1	2^0
$\frac{1}{2}$	$\frac{1}{2^1}$
$\frac{1}{4}$	$\frac{1}{2^2}$

Power, 3^n	Exponent
27	3^3
9	3^2
3	3^1
1	3^0
$\frac{1}{3}$	$\frac{1}{3^1}$
$\frac{1}{9}$	$\frac{1}{3^2}$

8.3 Guided Practice (pp. 503–505)

1. $\left(\frac{2}{3}\right)^0 = 1$
2. $(-8)^{-2} = \frac{1}{(-8)^2} = \frac{1}{64}$
3. $\frac{1}{2^{-3}} = \frac{1}{\frac{1}{2^3}} = \frac{1}{\frac{1}{8}} = 8$
4. $(-1)^0 = 1$
5. $\frac{1}{4^{-3}} = 4^3 = 64$
6. $(5^{-3})^{-1} = 5^{(-3)(-1)} = 5^3 = 125$
7. $(-3)^5 \cdot (-3)^{-5} = (-3)^{5+(-5)} = (-3)^0 = 1$
8. $\frac{6^{-2}}{6^2} = 6^{-2-2} = 6^{-4} = \frac{1}{6^4} = \frac{1}{1296}$
9. $\frac{3xy^{-3}}{9x^3y} = \frac{3x}{9x^3yy^3} = \frac{3x}{9x^3y^4} = \frac{1}{3x^2y^4}$

10. $10^{-27} \cdot 10^4 = 10^{-27+4} = 10^{-23}$

The order of magnitude of the mass of a proton is 10^{-23} gram.

8.3 Exercises (pp. 506–508)

Skill Practice

1. *Sample answer:* I would use the product of powers property because the expression simplifies to 3^0. By the definition of zero exponent, $3^0 = 1$.

2. *Sample answer:* The definition of negative exponents is defined only for nonzero bases.

3. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$ **4.** $7^{-3} = \frac{1}{7^3} = \frac{1}{343}$

5. $(-3)^{-1} = \frac{1}{(-3)^1} = -\frac{1}{3}$ **6.** $(-2)^{-6} = \frac{1}{(-2)^6} = \frac{1}{64}$

7. $2^0 = 1$ **8.** $(-4)^0 = 1$ **9.** $\left(\frac{3}{4}\right)^0 = 1$

10. $\left(-\frac{9}{16}\right)^0 = 1$ **11.** $\left(\frac{2}{7}\right)^{-2} = \frac{1}{\left(\frac{2}{7}\right)^2} = \frac{1}{\frac{4}{49}} = \frac{49}{4}$

12. $\left(\frac{4}{3}\right)^{-3} = \frac{1}{\left(\frac{4}{3}\right)^3} = \frac{1}{\frac{64}{27}} = \frac{27}{64}$

13. 0^{-3} = undefined **14.** 0^{-2} = undefined

15. $2^{-2} \cdot 2^{-3} = 2^{-2+(-3)} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

16. $7^{-6} \cdot 7^4 = 7^{-6+4} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

17. $(2^{-1})^5 = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

18. $(3^{-2})^2 = 3^{-2 \cdot 2} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

19. $\frac{1}{3^{-3}} = 3^3 = 27$ **20.** $\frac{1}{6^{-2}} = 6^2 = 36$

21. $\frac{3^{-3}}{3^2} = 3^{-3-2} = 3^{-5} = \frac{1}{3^5} = \frac{1}{243}$

22. $\frac{6^{-3}}{6^{-5}} = 6^{-3-(-5)} = 6^2 = 36$

23. $4\left(\frac{3}{2}\right)^{-1} = 4\left(\frac{3^{-1}}{2^{-1}}\right) = 4\left(\frac{2}{3}\right) = \frac{8}{3}$

24. $16\left(\frac{2^{-3}}{2^2}\right) = 16(2^{-3-2}) = 16(2^{-5}) = 16\left(\frac{1}{2^5}\right) = 16\left(\frac{1}{32}\right) = \frac{1}{2}$

25. $6^0 \cdot \left(\frac{1}{4^{-2}}\right) = 1 \cdot 4^2 = 16$

26. $3^{-2} \cdot \left(\frac{5}{7^0}\right) = \frac{1}{3^2} \cdot \left(\frac{5}{1}\right) = \frac{5}{9}$

27. 3^0 is 1, not 0.

$-6 \cdot 3^0 = -6 \cdot 1 = -6$

28. $x^{-4} = \frac{1}{x^4}$ **29.** $2y^{-3} = \frac{2}{y^3}$

30. $(4g)^{-3} = \frac{1}{(4g)^3} = \frac{1}{4^3g^3} = \frac{1}{64g^3}$

31. $(-11h)^{-2} = \frac{1}{(-11h)^2} = \frac{1}{(-11)^2h^2} = \frac{1}{121h^2}$

32. $x^2y^{-3} = \frac{x^2}{y^3}$ **33.** $5m^{-3}n^{-4} = \frac{5}{m^3n^4}$

34. $(6x^{-2}y^3)^{-3} = 6^{-3}x^{-2 \cdot (-3)}y^{3 \cdot (-3)} = \frac{1}{6^3}x^6y^{-9} = \frac{x^6}{216y^9}$

35. $(-15fg^2)^0 = 1$ **36.** $\frac{r^{-2}}{s^{-4}} = \frac{s^4}{r^2}$

37. $\frac{x^{-5}}{y^2} = \frac{1}{x^5y^2}$ **38.** $\frac{1}{8x^{-2}y^{-6}} = \frac{x^2y^6}{8}$

39. $\frac{1}{15x^{10}y^{-8}} = \frac{y^8}{15x^{10}}$ **40.** $\frac{1}{(-2z)^{-2}} = (-2z)^2 = 4z^2$

41. $\frac{9}{(3d)^{-3}} = 9(3d)^3 = 9 \cdot 27d^3 = 243d^3$

42. $\frac{(3x)^{-3}y^4}{-x^2y^{-6}} = \frac{y^4y^6}{-x^2(3x)^3} = \frac{y^4y^6}{-27x^2x^3} = \frac{y^{4+6}}{-27x^{2+3}} = -\frac{y^{10}}{27x^5}$

43. $\frac{12x^8y^{-7}}{(4x^{-2}y^{-6})^2} = \frac{12x^8y^{-7}}{4^2x^{-4}y^{-12}} = \frac{12x^8x^4y^{12}}{16y^7} = \frac{3x^{8+4}y^{12-7}}{4} = \frac{3x^{12}y^5}{4}$

44. D; $\frac{8}{4x^{-4}} = \frac{8x^4}{4} = 2x^4$

45. *D;* $(-4 \cdot 2^0 \cdot 3)^{-2} = (-12)^{-2} = \frac{1}{(-12)^2} = \frac{1}{144}$

46. False, $\frac{a^{-3}}{a^{-4}} = a^{-3-(-4)} = a^1$;

counterexample: $\frac{2^{-3}}{2^{-4}} = \frac{2^4}{2^3} = 2\left(\text{not } \frac{1}{2}\right)$

47. True, $\frac{a^{-1}}{b^{-1}} = \frac{\frac{1}{a}}{\frac{1}{b}} = \frac{b}{a}$

48. False, $a^{-1} + b^{-1} = \frac{1}{a} + \frac{1}{b} = \frac{b}{ab} + \frac{a}{ab} = \frac{b+a}{ab}$;

counterexample: $2^{-1} + 4^{-1} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}\left(\text{not } \frac{1}{6}\right)$

49. The value of a^{-n} as n increases approaches 0.

Problem Solving

50. $\frac{10^2}{10^{-4}} = 10^{2-(-4)} = 10^6$ grains of salt

51. $\frac{10^3}{10^{-2}} = 10^{3-(-2)} = 10^5$ grains of rice

52. $10^4 \text{ kg}\left(\frac{1000 \text{ g}}{1 \text{ kg}}\right) = 10^7 \text{ g}$

$\frac{10^7 \text{ g}}{10^{-4} \text{ g}} = 10^{7-(-4)} = 10^{11}$ times larger

53. $\frac{10^{-6} \text{ liter}}{10^7 \text{ red blood cells}} = \frac{10^{-2} \text{ liter}}{x}$

$10^{-6}x = 10^7 \cdot 10^{-2}$

$x = \frac{10^5}{10^{-6}}$

$x = 10^{5-(-6)} = 10^{11}$

The entire sample contained about 10^{11} red blood cells.

54. No; mass of giant fan palm $= 10^{-9} \cdot 10^{13} = 10^4$ g

$10^4\ \text{g}\left(\frac{1\ \text{kg}}{10^3\ \text{g}}\right) = 10$ kg, not 1 kg.

55. a.

Number of folds	0	1	2	3
Fraction of original area	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

b. $\left(\frac{1}{2}\right)^n$ where n is the number of folds

56. a. $t = \frac{(10^{-4})^2}{2(10^{-5})} = \frac{10^{-8}}{2(10^{-5})} = \frac{10^{-8+5}}{2} = \frac{10^{-3}}{2}$

$= \frac{1}{2} \times 10^{-3}$

$= 0.0005$ sec

b. $\frac{\text{cm}^2}{\frac{\text{cm}^2}{\text{sec}}} = \frac{\text{cm}^2 \cdot \text{sec}}{\text{cm}^2} = \text{sec}$

57. a. $I = 0.08Pd^{-2}$

$10^{-2} = 0.08P(30)^{-2}$

$\frac{10^{-2}}{(0.08)(30^{-2})} = P$

$\frac{30^2}{10^2(0.08)} = P$

112.5 watts $= P$

b. $I = 0.08(112.5)d^{-2} = \frac{9}{d^2}$

c. The intensity is divided by 4.

58. a. $\frac{1\ \text{lb}}{10^4\ \text{BTU}} = \frac{10\ \text{lb}}{x}$

$x = 10 \cdot 10^4$

$x = 10^5$

Your stereo uses 10^5 BTUs in 1 year.

b. $\frac{10^{-1}\ \text{lb}}{10^6\ \text{BTU}} = \frac{x}{10^5\ \text{BTU}}$

$10^6x = 10^{-1} \cdot 10^5$

$x = \frac{10^{-1} \cdot 10^5}{10^6}$

$x = 10^{-1+5-6}$

$x = 10^{-2} = 0.01$

0.01 pound of sulfur dioxide is added to the air.

Mixed Review

59. $10^3 \cdot 10^3 = 10^{3+3} = 10^6 = 1{,}000{,}000$

60. $10^2 \cdot 10^5 = 10^{2+5} = 10^7 = 10{,}000{,}000$

61. $\frac{10^9}{10^7} = 10^{9-7} = 10^2 = 100$

62. $\frac{10^6}{10^3} = 10^{6-3} = 10^3 = 1000$

63. $3x - 6 = -7x - 1$

$10x = 5$

$x = \frac{1}{2}$

$y = 3x - 6$

$y = 3\left(\frac{1}{2}\right) - 6$

$y = \frac{3}{2} - \frac{12}{2}$

$y = \frac{-9}{2}$

The solution is $\left(\frac{1}{2}, \frac{-9}{2}\right)$.

64. $-2x + 12 = -5x + 24$

$3x = 12$

$x = 4$

$y = -2x + 12$

$y = -2(4) + 12$

$y = 4$

The solution is (4, 4).

65. $-x + y = -8$

$y = x - 8$

$5x + (x - 8) = 40$

$6x - 8 = 40$

$6x = 48$

$x = 8$

$-x + y = -8$

$-8 + y = -8$

$y = 0$

The solution is (8, 0).

66. $-x - 2y = -6.5$

$-x = 2y - 6.5$

$x = -2y + 6.5$

$3(-2y + 6.5) - 6y = 16.5$

$-6y + 19.5 - 6y = 16.5$

$-12y = -3$

$y = 0.25$

$-x - 2(0.25) = -6.5$

$-x - 0.5 = -6.5$

$x = 6$

The solution is (6, 0.25).

67. $x = 2y + 5$

$3(2y + 5) + 4y = -5$

$6y + 15 + 4y = -5$

$10y = -20$

$y = -2$

$x = 2(-2) + 5 = -4 + 5 = 1$

The solution is (1, −2).

68. $2x + 6y = 5$

$-2x - 3y = 2$

$3y = 7$

$y = \frac{7}{3}$

$2x + 6\left(\frac{7}{3}\right) = 5$

$2x + 14 = 5$

$2x = -9$

$x = -\frac{9}{2}$

The solution is $\left(-\frac{9}{2}, \frac{7}{3}\right)$.

8.3 Extension (p. 510)

1. $100^{3/2} = 100^{(1/2) \cdot 3} = (100^{1/2})^3 = (\sqrt{100})^3 = 10^3 = 1000$

2. $121^{-1/2} = \frac{1}{121^{1/2}} = \frac{1}{\sqrt{121}} = \frac{1}{11}$

3. $81^{-3/2} = 81^{(1/2) \cdot (-3)} = (81^{1/2})^{-3} = (\sqrt{81})^{-3} = 9^{-3} = \frac{1}{9^3} = \frac{1}{729}$

4. $216^{2/3} = 216^{(1/3) \cdot 2} = (216^{1/3})^2 = (\sqrt[3]{216})^2 = 6^2 = 36$

5. $27^{-1/3} = \frac{1}{27^{1/3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

6. $343^{-2/3} = 343^{(1/3) \cdot (-2)} = (343^{1/3})^{-2} = (\sqrt[3]{343})^{-2} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

7. $9^{7/2} \cdot 9^{-3/2} = 9^{(7/2) + (-3/2)} = 9^{4/2} = 9^2 = 81$

8. $\left(\frac{1}{16}\right)^{1/2}\left(\frac{1}{16}\right)^{-1/2} = \left(\frac{1}{16}\right)^{(1/2) + (-1/2)} = \left(\frac{1}{16}\right)^0 = 1$

9. $36^{5/2} \cdot \frac{36^{-1/2}}{(36^{-1})^{-7/2}} = 36^{5/2} \cdot \frac{36^{-1/2}}{36^{7/2}}$

$= 36^{5/2} \cdot 36^{-1/2 - 7/2}$

$= 36^{5/2} \cdot 36^{-8/2}$

$= 36^{(5/2) + (-8/2)}$

$= 36^{-3/2}$

$= (36^{1/2})^{-3}$

$= 6^{-3}$

$= \frac{1}{6^3}$

$= \frac{1}{216}$

10. $(27^{-1/3})^3 = 27^{-3/3} = 27^{-1} = \frac{1}{27}$

11. $(-64)^{-5/3}(-64)^{4/3} = (-64)^{(-5/3) + (4/3)}$

$= (-64)^{-1/3}$

$= \frac{1}{(-64)^{1/3}}$

$= \frac{1}{\sqrt[3]{-64}}$

$= -\frac{1}{4}$

12. $(-8)^{1/3}(-8)^{-2/3}(-8)^{1/3} = (-8)^{(1/3) + (-2/3) + (1/3)}$

$= (-8)^0 = 1$

13. $b^3 = a$

$(a^k)^3 = a$

$a^{3k} = a^1$

$3k = 1$

$k = \frac{1}{3}$

Mixed Review of Problem Solving (p. 511)

1. $\frac{10^{12}}{10^9} = 10^{12 - 9} = 10^3 = 1000$ times faster

2. a. $V = s^3 = \left(\frac{9}{2}\right)^3 = \frac{9^3}{2^3} = \frac{729}{8}$ in.^{3}

b. Power of a quotient property.

3. a. Because $V = \frac{4}{3}\pi r^3$, the order of magnitude of the volume of the droplet is $(1)(1)(10^{-4})^3 = 10^{-12}$ cm^3.

b. $r = 10^{-4} \cdot 10^2 = 10^{-4+2} = 10^{-2}$

Because $V = \frac{4}{3}\pi r^3$, the order of magnitude of the volume of the raindrop is $(1)(1)(10^{-2})^3 = 10^{-6}$ cm^3.

c. Divide the volume of the raindrop by the volume of the droplet.

$\frac{10^{-6}}{10^{-12}} = 10^{-6 - (-12)} = 10^6$ droplets

Quotient of powers property

4. $10^{-12} \cdot 10^{15} = 10^{-12 + 15} = 10^3 = 1000 \frac{\text{watts}}{\text{m}^2}$

5. a. $\frac{10^{-1}}{10^5} = 10^{-1-5} = 10^{-6}$ in.

$\frac{\text{in.}^3}{\text{in.}^2} = \text{in.}$

b. $10^7 \cdot 10^{-6} = 10^{7 + (-6)} = 10$ in.^{3}

c. $10^x \cdot 10^{-6} = 10^{x + (-6)} = 10^{x-6}$ in.^{3}

6. a. *Sample answer:* How many milliseconds are in a gigasecond?

b. *Sample answer:* How many megaseconds are in a gigasecond?

Lesson 8.4

8.4 Guided Practice (pp. 512–514)

1. $539{,}000 = 5.39 \times 10^5$

$4.5 \times 10^{-4} = 0.00045$

2. $27{,}500 = 2.75 \times 10^4$

$2.75 \times 10^4 < 3.401 \times 10^4 < 2.7 \times 10^5$

order: 27,500; 3.401×10^4; 2.7×10^5

3. $(1.3 \times 10^{-5})^2 = 1.3^2 \times (10^{-5})^2 = 1.69 \times 10^{-10}$

4. $\frac{4.5 \times 10^5}{1.5 \times 10^{-2}} = \frac{4.5}{1.5} \times \frac{10^5}{10^{-2}} = 3 \times 10^7$

5. $(1.1 \times 10^7)(4.2 \times 10^2) = (1.1 \cdot 4.2) \times (10^7 \cdot 10^2)$

$= 4.62 \times 10^9$

6. $\frac{5.0 \times 10^{-1}}{5.0 \times 10^{-3}} = \frac{5.0}{5.0} \times \frac{10^{-1}}{10^{-3}} = 1 \times 10^2 = 100$

The radius of the arteriole is 100 times the radius of the capillary.

$100^2 = 10{,}000 = 10^4$

The cross-sectional area of the arteriole is 10^4 times the cross-sectional area of the capillary.

8.4 Exercises (pp. 512–518)

Skill Practice

1. No; 0.5 is not greater than or equal to 1.0 and less than 10.

2. Greater than 1; the exponent is positive.

3. $8.5 = 8.5 \times 10^0$

4. $0.72 = 7.2 \times 10^{-1}$

5. $82.4 = 8.24 \times 10^1$

6. $0.005 = 5 \times 10^{-3}$

7. $72{,}000{,}000 = 7.2 \times 10^7$

8. $0.00406 = 4.06 \times 10^{-3}$

9. $1{,}065{,}250 = 1.06525 \times 10^6$

10. $0.000045 = 4.5 \times 10^{-5}$

11. $1{,}060{,}000{,}000 = 1.06 \times 10^9$

12. $0.00000526 = 5.26 \times 10^{-6}$

13. $900{,}000{,}000{,}000{,}000 = 9 \times 10^{14}$

14. $0.00000007008 = 7.008 \times 10^{-8}$

15. C; $54{,}004{,}000{,}000 = 5.4004 \times 10^{10}$

16. $2.6 \times 10^3 = 2600$

17. $7.5 \times 10^7 = 75{,}000{,}000$

18. $1.11 \times 10^2 = 111$

19. $3.03 \times 10^4 = 30{,}300$

20. $4.709 \times 10^6 = 4{,}709{,}000$

21. $1.544 \times 10^{10} = 15{,}440{,}000{,}000$

22. $6.1 \times 10^{-3} = 0.0061$

23. $4.4 \times 10^{-10} = 0.00000000044$

24. $2.23 \times 10^{-6} = 0.00000223$

25. $8.52 \times 10^{-8} = 0.0000000852$

26. $6.4111 \times 10^{-10} = 0.00000000064111$

27. $1.2034 \times 10^{-6} = 0.0000012034$

28. The error is that the decimal point was moved 3 places to the right instead of 3 places to the left.

$1.24 \times 10^{-3} = 0.00124$

29. $45{,}000 = 4.5 \times 10^4$

$12{,}439 = 1.2439 \times 10^4$

$6.7 \times 10^3 < 1.2439 \times 10^4 < 2 \times 10^4 < 4.5 \times 10^4$

Order: 6.7×10^3; 12,439; 2×10^4; 45,000

30. $65{,}000{,}000 = 6.5 \times 10^7$

$55{,}004{,}000 = 5.5004 \times 10^7$

$6.07 \times 10^6 < 6.2 \times 10^6 < 3.557 \times 10^7 < 5.5004 \times 10^7 < 6.5 \times 10^7$

Order: 6.07×10^6; 6.2×10^6; 3.557×10^7; 55,004,000; 65,000,000

31. $0.0005 = 5 \times 10^{-4}$

$0.00008 = 8 \times 10^{-5}$

$0.04065 = 4.065 \times 10^{-2}$

$9.8 \times 10^{-6} < 8 \times 10^{-5} < 5 \times 10^{-4} < 5 \times 10^{-3} < 8.2 \times 10^{-3} < 4.065 \times 10^{-2}$

Order: 9.8×10^{-6}; 0.00008; 0.0005; 5×10^{-3}; 8.2×10^{-3}; 0.04065

32. $0.0000395 = 3.95 \times 10^{-5}$

$0.00010068 = 1.0068 \times 10^{-4}$

$0.000005 = 5 \times 10^{-6}$

$5 \times 10^{-6} < 5.08 \times 10^{-6} < 2.4 \times 10^{-5} < 3.95 \times 10^{-5} < 1.0068 \times 10^{-4}$

Order: 0.000005; 5.08×10^{-6}; 2.4×10^{-5}; 0.0000395; 0.00010068

33. $56{,}000 = 5.6 \times 10^4$

$5.6 \times 10^3 < 56{,}000$

34. $404{,}000.1 = 4.040001 \times 10^5$

$404{,}000.1 < 4.04001 \times 10^5$

35. $0.00986 = 9.86 \times 10^{-3}$

$9.86 \times 10^{-3} = 0.00986$

36. $0.003309 = 3.309 \times 10^{-3}$

37. $0.0000203 = 2.03 \times 10^{-5}$

$2.203 \times 10^{-4} > 0.0000203$

38. $604{,}589{,}000 = 6.04589 \times 10^8$

$604{,}589{,}000 > 6.04589 \times 10^7$

39. $(4.4 \times 10^3)(1.5 \times 10^{-7}) = (4.4 \cdot 1.5) \times (10^3 \cdot 10^{-7})$

$= 6.6 \times 10^{-4}$

40. $(7.3 \times 10^{-5})(5.8 \times 10^2) = (7.3 \cdot 5.8) \times (10^{-5} \cdot 10^2)$

$= 42.34 \times 10^{-3}$

$= (4.234 \times 10^1) \times 10^{-3}$

$= 4.234 \times (10^1 \cdot 10^{-3})$

$= 4.234 \times 10^{-2}$

41. $(8.1 \times 10^{-4})(9 \times 10^{-6}) = (8.1 \cdot 9) \times (10^{-4} \cdot 10^{-6})$

$= 72.9 \times 10^{-10}$

$= (7.29 \times 10^1) \times 10^{-10}$

$= 7.29 \times (10^1 \cdot 10^{-10})$

$= 7.29 \times 10^{-9}$

42. $\frac{6 \times 10^{-3}}{8 \times 10^{-6}} = \frac{6}{8} \times \frac{10^{-3}}{10^{-6}}$

$= 0.75 \times 10^3$

$= (7.5 \times 10^{-1}) \times 10^3$

$= 7.5 \times (10^{-1} \cdot 10^3)$

$= 7.5 \times 10^2$

43. $\frac{5.4 \times 10^{-5}}{1.8 \times 10^{-2}} = \frac{5.4}{1.8} \times \frac{10^{-5}}{10^{-2}} = 3 \times 10^{-3}$

44. $\frac{4.1 \times 10^4}{8.2 \times 10^8} = \frac{4.1}{8.2} \times \frac{10^4}{10^8}$

$= 0.5 \times 10^{-4}$

$= (5 \times 10^{-1}) \times 10^{-4}$

$= 5 \times (10^{-1} \cdot 10^{-4})$

$= 5 \times 10^{-5}$

45. $(5 \times 10^{-8})^3 = (5)^3 \times (10^{-8})^3$

$= 125 \times 10^{-24}$

$= (1.25 \times 10^2) \times 10^{-24}$

$= 1.25 \times (10^2 \cdot 10^{-24})$

$= 1.25 \times 10^{-22}$

46. $(7 \times 10^{-5})^4 = (7)^4 \times (10^{-5})^4$

$= 2401 \times 10^{-20}$

$= (2.401 \times 10^3) \times 10^{-20}$

$= 2.401 \times (10^3 \cdot 10^{-20})$

$= 2.401 \times 10^{-17}$

47. $(1.4 \times 10^3)^2 = (1.4)^2 \times (10^3)^2 = 1.96 \times 10^6$

48. B;

$\frac{1.235 \times 10^4}{9.5 \times 10^7} = \frac{1.235}{9.5} \times \frac{10^4}{10^7}$

$= 0.13 \times 10^{-3}$

$= (1.3 \times 10^{-1}) \times 10^{-3}$

$= 1.3 \times (10^{-1} \cdot 10^{-3})$

$= 1.3 \times 10^{-4}$

49. *Sample answer:* 2.8×10^1 and 1×10^3; 11.2×10^5 and 4.0×10^1

50. *Sample answer:*

Step 1: Write the numbers so that they have the same exponent.

$6.7 \times 10^4 = 0.67 \times 10^5$

3.6×10^5

Step 2: Add the numbers together by adding the non-exponential parts of each number together.

$0.67 \times 10^5 + 3.6 \times 10^5 = (3.6 + 0.67) \times 10^5$

$= 4.27 \times 10^5$

Problem Solving

51.

Insect	Length (mm)
Fringed ant beetle	2.5×10^{-1}
Walking stick	$555 = 5.55 \times 10^2$
Parasite wasp	1.4×10^{-4}
Elephant beetle	1.67×10^2

a. $1.4 \times 10^{-4} < 2.5 \times 10^{-1} < 1.67 \times 10^2 < 5.55 \times 10^2$

The lengths from least to greatest are:

1.4×10^{-4}; 2.5×10^{-1}; 1.67×10^2; 555

b. The elephant beetle and the walking stick are longer than the fringed ant beetle.

52. *Voyager 1*: $9{,}643{,}000{,}000 = 9.643 \times 10^9$ miles

Voyager 2: 9.065×10^9 miles

$9.065 \times 10^9 < 9.643 \times 10^9$

So, *Voyager 1* traveled the greater distance.

53. Cotton per acre $= \frac{9.7 \times 10^8}{6.9 \times 10^5}$

$= \frac{9.7}{6.9} \times \frac{10^8}{10^5}$

$\approx 1.4058 \times 10^3$

$= 1405.8$

≈ 1406 lb per acre

54. $\frac{\text{Amazon}}{\text{Mississippi}} = \frac{7.6 \times 10^6}{5.53 \times 10^5}$

$= \frac{7.6}{5.53} \times \frac{10^6}{10^5}$

$\approx 1.37 \times 10^1$

$= 13.7$

≈ 14

The ratio tells you that the Amazon is flowing about 14 times as fast as the Mississippi River.

55. a. $\frac{\text{Earth}}{\text{moon}} = \frac{6.38 \times 10^3}{1.74 \times 10^3} = \frac{6.38}{1.74} \times \frac{10^3}{10^3} \approx 3.67$

The radius of Earth is about 3.67 times the radius of the moon.

b. Earth:

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(6.38 \times 10^3)^3$

$= \frac{4}{3}\pi(6.38^3) \times (10^3)^3$

$= \frac{4}{3}\pi(259.69 \times 10^9)$

$= \frac{4}{3}\pi(2.5969 \times 10^2 \times 10^9)$

$= \frac{4}{3}\pi(2.5969 \times 10^{11})$

Moon:

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(1.74 \times 10^3)^3$

$= \frac{4}{3}\pi(1.74)^3 \times (10^3)^3$

$= \frac{4}{3}\pi(5.268 \times 10^9)$

$\frac{\text{Earth}}{\text{Moon}} = \frac{\frac{4}{3}\pi(2.5969 \times 10^{11})}{\frac{4}{3}\pi(5.268 \times 10^9)}$

$= \frac{2.5969}{5.268} \times \frac{10^{11}}{10^9}$

$\approx 0.49296 \times 10^2$

$= 4.9296 \times 10^{-1} \times 10^2$

$= 4.9296 \times 10^1$

≈ 49.30

The volume of Earth is about 49.30 times the volume of the moon.

c. The ratio of the volume is the cube of the ratio of the radii.

56. a. $n = (5 \times 10^7) \times 200$

$= (5 \times 10^7) \times (2 \times 10^2)$

$= (5 \times 2) \times (10^7 \cdot 10^2)$

$= 10 \times 10^9$

$= (1 \times 10^1) \times 10^9$

$= 1 \times (10^1 \cdot 10^9)$

$= 1 \times 10^{10}$ locusts

b. $m = (1 \times 10^{10}) \times (2 \times 10^0)$

$= (1 \times 2) \times (10^{10} \cdot 10^0)$

$= 2 \times 10^{10}\text{g}$

$= 2 \times 10^{10}\text{g} \cdot \frac{1 \text{ kg}}{1 \times 10^3 \text{ g}}$

$= \frac{2}{1} \times \frac{10^{10}}{10^3}$

$= 2 \times 10^7$ kg

57. 1 pixel $= 4 \times 10^{-3}$ in.

$(1 \times 10^3) \cdot (4 \times 10^{-3}) = (1 \cdot 4) \times (10^3 \cdot 10^{-3})$

$= 4 \times 10^0$

$= 4$

$(1.5 \times 10^3) \cdot (4 \times 10^{-3}) = (1.5 \cdot 4) \times (10^3 \cdot 10^{-3})$

$= 6 \times 10^0$

$= 6$

The dimensions of the print are 4 inches by 6 inches.

58. a. Speed of light $= \frac{1.863 \times 10^5 \text{ mi}}{\text{sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}$

$\cdot \frac{24 \text{ h}}{1 \text{ day}} \cdot \frac{365 \text{ days}}{1 \text{ yr}} \approx \frac{5.8752 \times 10^{12} \text{ min}}{\text{yr}}$

b.

Years	Distance (mi)
$1 = 10^0$	5.875×10^{12}
$10 = 10^1$	5.875×10^{13}
$100 = 10^2$	5.875×10^{14}
$1000 = 10^3$	5.875×10^{15}
$10{,}000 = 10^4$	5.875×10^{16}
$100{,}000 = 10^5$	5.875×10^{17}

It would take 10^5, or 100,000, years for light to travel across our galaxy.

59. a. $70(7 \times 10^{-2}) = 490 \times 10^{-2}$

$= 4.9 \times 10^2 \times 10^{-2}$

$= 4.9$ L per min

b. $\frac{4.9\text{L}}{\text{min}} \times \frac{5.265 \times 10^5 \text{ min}}{1 \text{ yr}} = 25.7985 \times 10^5$

$= 2.57985 \times 10^1 \times 10^5$

$= 2.57985 \times 10^6$ L per yr

$10(2.57985 \times 10^6) = 2.57985 \times 10^7$ L per 10 yr

$8(2.57985 \times 10^7) = (8 \times 2.57985) \times 10^7$

$= 20.6388 \times 10^7$

$= 2.06388 \times 10^1 \times 10^7$

$= 2.06388 \times 10^8$ L per 80 yr

c. Underestimates; *Sample answer*: They are calculated when a person is at rest. When a person is not resting, the rate will go up.

60. a. $\frac{X45}{C9} = \frac{45 \times 10^{-4}}{9 \times 10^{-6}} = \frac{45}{9} \times \frac{10^{-4}}{10^{-6}} = 5 \times 10^2$

$= 500$ times greater

b. time $= \frac{\text{distance}}{\text{rate}} = \frac{1.5 \times 10^{11} \text{ m}}{8.2 \times 10^6 \text{ km/h}}$

$= 1.83 \times 10^1 = 18.3$ h

Mixed Review

61. $33\% = \frac{33}{100} = 0.33$

62. $62.7\% = \frac{62.7}{100} = 0.627$

63. $0.9\% = \frac{0.9}{100} = 0.009$

64. $0.04\% = \frac{0.4}{100} = 0.0004$

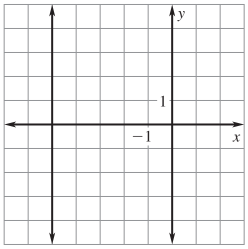
65. $3.95\% = \frac{3.95}{100} = 0.0395$

66. $\frac{1}{4}\% = \frac{0.25}{100} = 0.0025$

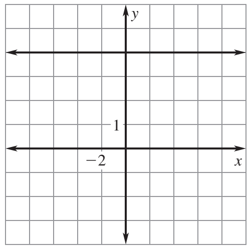
67. $\frac{5}{2}\% = \frac{2.5}{100} = 0.025$

68. $133\% = \frac{133}{100} = 1.33$

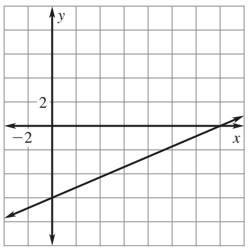
69. $x = -5$



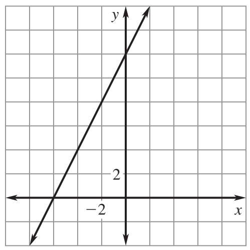
70. $y = 4$



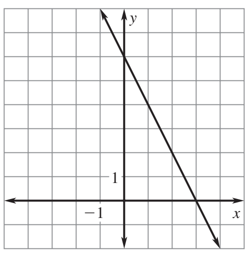
71. $3x - 7y = 42$



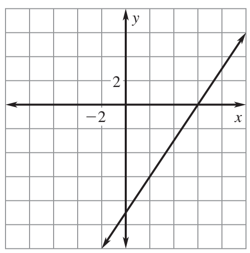
72. $y - 2x = 12$



73. $y = -2x + 6$



74. $y = 1.5x - 9$



Quiz 8.3–8.4 (p. 518)

1. $(-4x)^4 \cdot (-4)^{-6} = (-4)^4 \cdot x^4 \cdot (-4)^{-6}$

$= (-4)^{4 + (-6)} \cdot x^4$

$= (-4)^{-2}x^4$

$= \dfrac{x^4}{(-4)^2}$

$= \dfrac{x^4}{16}$

2. $(-3x^7y^{-2})^{-3} = (-3)^{-3} \cdot (x^7)^{-3} \cdot (y^{-2})^{-3}$

$= (-3)^{-3} \cdot x^{-21} \cdot y^6$

$= \dfrac{y^6}{(-3)^3 \cdot x^{21}}$

$= -\dfrac{y^6}{27x^{21}}$

3. $\dfrac{1}{(5z)^{-3}} = (5z)^3 = 5^3 \cdot z^3 = 125z^3$

4. $\dfrac{(6x)^{-2}y^5}{-x^3y^{-7}} = \dfrac{6^{-2}x^{-2}y^5}{-x^3y^{-7}} = \dfrac{y^5y^7}{-6^2x^2x^3} = -\dfrac{y^{12}}{36x^5}$

5. $6.02 \times 10^6 = 6{,}020{,}000$

6. $5.41 \times 10^{11} = 541{,}000{,}000{,}000$

7. $8.007 \times 10^{-5} = 0.00008007$

8. $9.253 \times 10^{-7} = 0.0000009253$

9.

Dinosaur	Mass (kilograms)
Brachiosaurus	$77{,}100 = 7.71 \times 10^4$
Diplodocus	1.06×10^4
Apatosaurus	$29{,}900 = 2.99 \times 10^4$
Ultrasaurus	1.36×10^5

a. $1.06 \times 10^4 < 2.99 \times 10^4 < 7.71 \times 10^4 < 1.36 \times 10^5$

order: 1.06×10^4; 29,900; 77,100; 1.36×10^5

b. Ultrasaurus is more massive than Brachiosaurus.

Graphing Calculator Activity 8.4 (p. 519)

1. $(1.5 \times 10^4)(1.8 \times 10^9) = 2.7\text{E}13 = 2.7 \times 10^{13}$

2. $(2.6 \times 10^{-14})(1.4 \times 10^{20}) = 3.64\text{E}6 = 3.64 \times 10^6$

3. $(7.0 \times 10^{25}) \div (2.8 \times 10^6) = 2.5\text{E}19 = 2.5 \times 10^{19}$

4. $(4.5 \times 10^{15}) \div (9.0 \times 10^{-2}) = 5\text{E}16 = 5 \times 10^{16}$

5. a. grams used = grams used per gallon • number of gallons

$= (4.45 \times 10^7) \cdot (1.37 \times 10^{11})$

$\approx 6.10 \times 10^{18}$

About 6.10×10^{18} grams of carbon were used to produce the gasoline.

b. atoms of carbon = atoms per 1 gram of carbon • grams of carbon

$\approx (5.0 \times 10^{22}) \cdot (6.10 \times 10^{18})$

$= 3.05 \times 10^{41}$

About 3.05×10^{41} atoms of carbon were used.

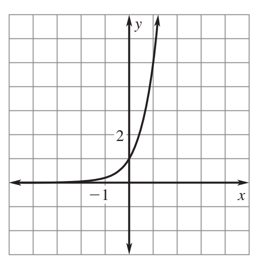
Lesson 8.5

8.5 Guided Practice (pp. 520–523)

1. The y-values are multiplied by 3 for each increase of 1 in x, so $y = a \cdot 3^x$. When $x = 0$, $y = 27 = a$. So, a rule for the function is $y = 27 \cdot 3^x$.

2. $y = 5^x$

x	−2	−1	0	1	2
y	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25

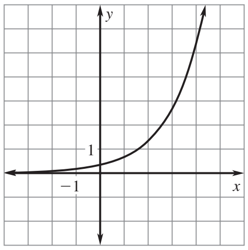


Domain: all real numbers

Range: all positive real numbers

3. $y = \frac{1}{3} \cdot 2^x$

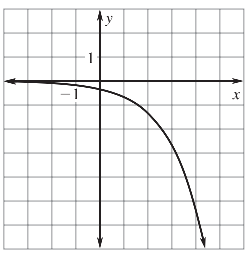
x	−2	−1	0	1	2
y	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{3}$



The graph of $y = \frac{1}{3} \cdot 2^x$ is a vertical shrink of the graph of $y = 2^x$.

4. $y = -\frac{1}{3} \cdot 2^x$

x	−2	−1	0	1	2
y	$-\frac{1}{12}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$



The graph of $y = -\frac{1}{3} \cdot 2^x$ is a vertical shrink with a reflection in the x-axis of the graph of $y = 2^x$.

5. $C = 11{,}000(1.069)^t = 11{,}000(1.069)^{10} \approx 21{,}437$

   In 1994, the value of the car was about $21,437.

6. $y = a(1 + r)^t = 250(1 + 0.035)^5 = 250(1.035)^5$

   ≈ 296.92

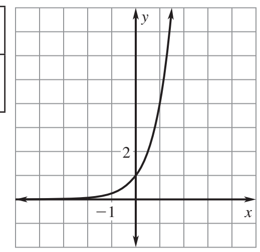
   You will have $296.92 in 5 years.

8.5 Exercises (pp. 523–527)

Skill Practice

1. In the exponential growth model $y = a(1 + r)^t$, the quantity $1 + r$ is called the *growth factor.*
2. The exponential function $y = ab^x$ (where $a > 0$) represents exponential growth when $b > 1$.
3. $y = 2 \cdot 5^x$ is a vertical stretch of $y = 5^x$. The y-values for $y = 2 \cdot 5^x$ are 2 times the corresponding y-values for $y = 5^x$.
4. The y-values are multiplied by 2 for each increase of 1 in x, so $y = a \cdot 2^x$. When $x = 0$, $y = 4 = a$. So, a rule for the function is $y = 4 \cdot 2^x$.
5. The y-values are multiplied by 5 for each increase of 1 in x, so $y = a \cdot 5^x$. When $x = 0$, $y = 125 = a$. So, a rule for the function is $y = 125 \cdot 5^x$.
6. The y-values are multiplied by 2 for each increase of 1 in x, so $y = a \cdot 2^x$. When $x = 0$, $y = \frac{1}{2} = a$. So, a rule for the function is $y = \frac{1}{2} \cdot 2^x$.
7. The y-values are multiplied by 3 for each increase of 1 in x, so $y = a \cdot 3^x$. When $x = 0$, $y = \frac{1}{9} = a$. So, a rule for the function is $y = \frac{1}{9} \cdot 3^x$.
8. *Sample answer:* If the function is linear, for each increase of 1 in x, the corresponding y-values increase by a set amount. If the function is exponential, for each increase of 1 in x, the corresponding y-values are multiplied by a set amount.
9. $y = 4^x$

x	−2	−1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

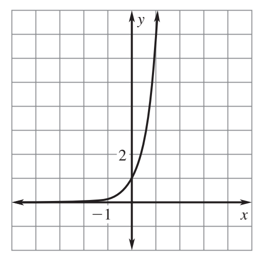


Domain: All real numbers

Range: $y > 0$

10. $y = 7^x$

x	−3	−2	−1	0	1
y	$\frac{1}{343}$	$\frac{1}{49}$	$\frac{1}{7}$	1	7

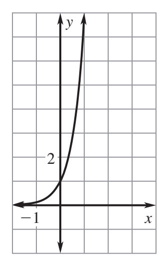


Domain: All real numbers

Range: $y > 0$

11. $y = 8^x$

x	−3	−2	−1	0	1
y	$\frac{1}{512}$	$\frac{1}{64}$	$\frac{1}{8}$	1	8

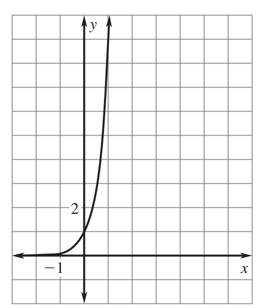


Domain: All real numbers

Range: $y > 0$

12. $y = 9^x$

x	−3	−2	−1	0	1
y	$\frac{1}{729}$	$\frac{1}{81}$	$\frac{1}{9}$	1	9

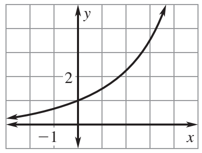


Domain: All real numbers

Range: $y > 0$

13. $y = (1.5)^x$

x	−2	−1	0	1	2
y	0.44	0.67	1	1.5	2.25

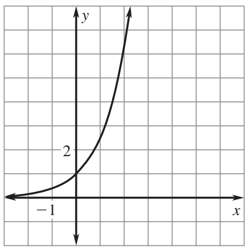


Domain: All real numbers

Range: $y > 0$

14. $y = (2.5)^x$

x	−2	−1	0	1	2
y	0.16	0.4	1	2.5	6.25

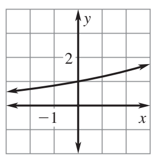


Domain: All real numbers

Range: $y > 0$

15. $y = (1.2)^x$

x	−1	0	1	2	3
y	0.83	1	1.2	1.44	1.728

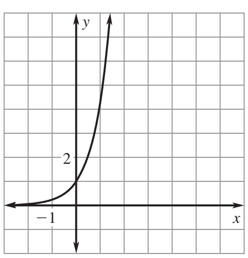


Domain: All real numbers

Range: $y > 0$

16. $y = (4.3)^x$

x	−3	−2	−1	0	1
y	0.01	0.05	0.23	1	4.3

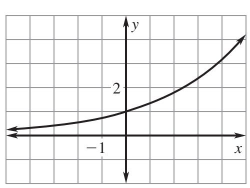


Domain: All real numbers

Range: $y > 0$

17. $y = \left(\frac{4}{3}\right)^x$

x	−1	0	1	2	3
y	$\frac{3}{4}$	1	$\frac{4}{3}$	$\frac{16}{9}$	$\frac{64}{27}$

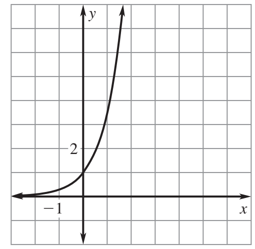


Domain: All real numbers

Range: $y > 0$

18. $y = \left(\frac{7}{2}\right)^x$

x	−2	−1	0	1	2
y	$\frac{4}{49}$	$\frac{2}{7}$	1	$\frac{7}{2}$	$\frac{49}{4}$

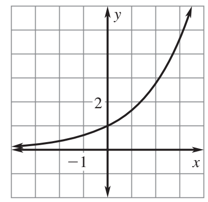


Domain: All real numbers

Range: $y > 0$

19. $y = \left(\frac{5}{3}\right)^x$

x	−1	0	1	2	3
y	$\frac{3}{5}$	1	$\frac{5}{3}$	$\frac{25}{9}$	$\frac{125}{27}$

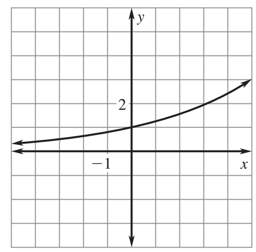


Domain: All real numbers

Range: $y > 0$

20. $y = \left(\frac{5}{4}\right)^x$

x	−1	0	1	2	3
y	$\frac{4}{5}$	1	$\frac{5}{4}$	$\frac{25}{16}$	$\frac{125}{64}$



Domain: All real numbers

Range: $y > 0$

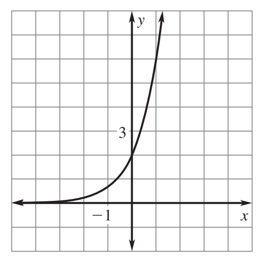
21. The error is that the percent increase was not written as a decimal.

$P = a(1 + r)^t = 0.27(1 + 0.02)^3 = 0.27(1.02)^3 = 0.29$

In 2002 the price of a pound of flour was $.29.

22. $y = 2 \cdot 3^x$

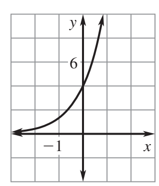
x	−2	−1	0	1	2
y	$\frac{2}{9}$	$\frac{2}{3}$	2	6	18



The graph of $y = 2 \cdot 3^x$ is a vertical stretch of the graph of $y = 3^x$.

23. $y = 4 \cdot 3^x$

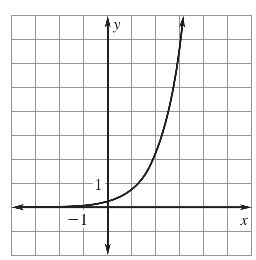
x	−3	−2	−1	0	1
y	$\frac{4}{27}$	$\frac{4}{9}$	$\frac{4}{3}$	4	12



The graph of $y = 4 \cdot 3^x$ is a vertical stretch of the graph of $y = 3^x$.

24. $y = \frac{1}{4} \cdot 3^x$

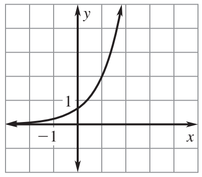
x	−1	0	1	2	3
y	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{9}{4}$	$\frac{27}{4}$



The graph of $y = \frac{1}{4} \cdot 3^x$ is a vertical shrink of the graph of $y = 3^x$.

25. $y = \frac{2}{3} \cdot 3^x$

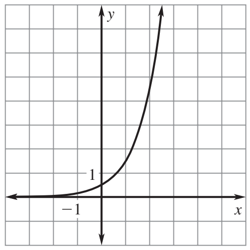
x	−2	−1	0	1	2
y	$\frac{2}{27}$	$\frac{2}{9}$	$\frac{2}{3}$	2	6



The graph of $y = \frac{2}{3} \cdot 3^x$ is a vertical shrink of the graph of $y = 3^x$.

26. $y = 0.5 \cdot 3^x$

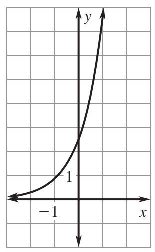
x	−2	−1	0	1	2
y	$\frac{1}{18}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{2}$



The graph of $y = (0.5) \cdot 3^x$ is a vertical shrink of the graph of $y = 3^x$.

27. $y = 2.5 \cdot 3^x$

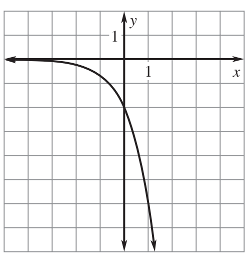
x	−3	−2	−1	0	1
y	0.09	0.28	0.83	2.5	7.5



The graph of $y = 2.5 \cdot 3^x$ is a vertical stretch of the graph of $y = 3^x$.

28. $y = -2 \cdot 3^x$

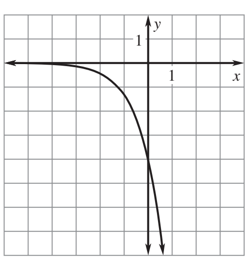
x	−2	−1	0	1	2
y	$-\frac{2}{9}$	$-\frac{2}{3}$	−2	−6	−18



The graph of $y = -2 \cdot 3^x$ is a vertical stretch with a reflection in the x-axis of the graph of $y = 3^x$.

29. $y = -4 \cdot 3^x$

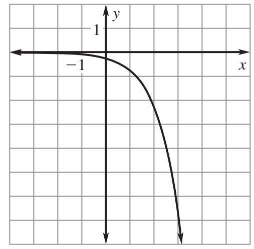
x	−3	−2	−1	0	1
y	$-\frac{4}{27}$	$-\frac{4}{9}$	$-\frac{4}{3}$	−4	−12



The graph of $y = -4 \cdot 3^x$ is a vertical stretch with a reflection in the x-axis of the graph of $y = 3^x$.

30. $y = -\frac{1}{4} \cdot 3^x$

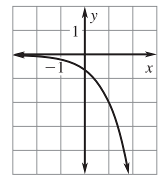
x	−2	−1	0	1	2
y	$-\frac{1}{36}$	$-\frac{1}{12}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{9}{4}$



The graph of $y = -\frac{1}{4} \cdot 3^x$ is a vertical shrink with a reflection in the x-axis of the graph of $y = 3^x$.

31. $y = -\frac{2}{3} \cdot 3^x$

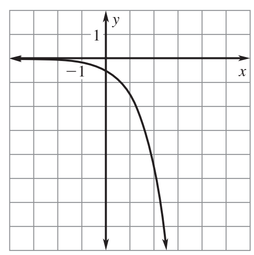
x	−2	−1	0	1	2
y	$-\frac{2}{27}$	$-\frac{2}{9}$	$-\frac{2}{3}$	−2	−6



The graph of $y = -\frac{2}{3} \cdot 3^x$ is a vertical shrink with a reflection in the x-axis of the graph of $y = 3^x$.

32. $y = -0.5 \cdot 3^x$

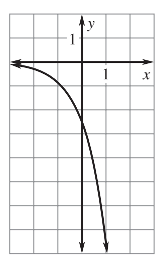
x	−2	−1	0	1	2
y	$-\frac{1}{18}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{9}{2}$



The graph of $y = -\frac{1}{2} \cdot 3^x$ is a vertical shrink with a reflection in the x-axis of the graph of $y = 3^x$.

33. $y = -2.5 \cdot 3^x$

x	−3	−2	−1	0	1
y	−0.09	−0.28	−0.83	−2.5	−7.5



The graph of $y = -2.5 \cdot 3^x$ is a vertical stretch with a reflection in the x-axis of the graph of $y = 3^x$.

34. C;

$f(x) = \frac{1}{3} \cdot 6^x$

$\frac{1}{3} \stackrel{?}{=} \frac{1}{3} \cdot 6^{(0)}$

$\frac{1}{3} = \frac{1}{3}$

$f(x) = \frac{1}{3} \cdot 6^x$

$2 \stackrel{?}{=} \frac{1}{3} \cdot 6^{(1)}$

$2 \stackrel{?}{=} \frac{1}{3} \cdot 6$

$2 = 2$

35. 200%; *Sample answer*: A growth rate of 200% would create a growth factor of $1 + 2 = 3$, which would represent the tripling of the population every year.

36. Linear:

Slope $= \frac{6 - 2}{1 - 0} = 4$

Point-slope form: $y - 2 = 4(x - 0)$

$y = 4x + 2$

Exponential: An increase by 1 in x means multiplication by 3 in y, so $y = a \cdot 3^x$.

When $x = 0$, $y = 2 = a$.

So, $y = 2 \cdot 3^x$.

37. The graphs are the same.

$f(x) = 2^{x+2} = 2^x \cdot 2^2 = 2^x \cdot 4 = 4 \cdot 2^x = g(x)$

Problem Solving

38. $y = a(1 + r)^t$

a. $y = (125)(1 + 0.05)^1 = \$131.25$

b. $y = (125)(1 + 0.05)^2 = \$137.81$

c. $y = (125)(1 + 0.05)^5 = \$159.54$

d. $y = (125)(1 + 0.05)^{20} = \$331.66$

39. a. $r = 10\% = 0.1$

$a = 600$ (million)

$c = 600(1 + 0.1)^t$

$c = 600(1.1)^t$, where c is the number of computers (in millions) and t is the number of years since 2001.

b. $c = 600(1.1)^t = 600(1.1)^8 = 1286.153286$

There will be about 1,286,153,286 computers in use worldwide in 2009.

40. a. $r = 7\% = 0.07$

$a = 3{,}173{,}000$

$g = 3{,}173{,}000(1 + 0.07)^t$

$g = 3{,}173{,}000(1.07)^t$, where g is the number of gas grills and t is the number of years since 1985.

b. $g = 3{,}173{,}000(1.07)^t = 3{,}173{,}000(1.07)^{17}$

$= 10{,}022{,}920.66$

About 10,022,921 gas grills were shipped in 2002.

41. a. Tree 1:

$A = 154(1 + 0.06)^t$

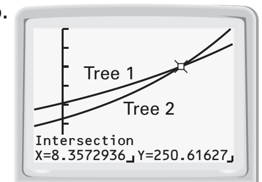
$A = 154(1.06)^t$

Tree 2:

$A = 113(1 + 0.1)^t$

$A = 113(1.1)^t$

b.



In about 8.4 years the trees will have the same basal area.

42. Yes; *Sample answer:* For each increase of 5 feet in length, the cost is multiplied by $\frac{7}{4}$.

43. C;

Time	Blogs
0 months	600,000
6 months	1,200,000
12 months	2,400,000
18 months	4,800,000

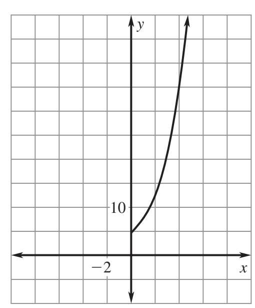
44. a. initial amount = 4.67 million

growth factor = 1.65

growth rate = 0.65

b.

x	0	1	2	3	4
y	4.67	7.71	12.71	20.98	34.61



Domain: $0 \le x \le 10$

Range: 4.67 million $\le y \le$ 698.48 million

c. When $x = 3$, or in 1994, the number of Internet users worldwide was about 21 million.

45. $y = 25.96(1.059)^x$

When $x = 30$, $y \approx 145$ hertz.

Chapter 8

46. **a.** growth per year $= \dfrac{62{,}947{,}714 - 12{,}866{,}020}{1890 - 1830}$

$= \dfrac{50{,}081{,}694}{60}$

$= 834{,}694.9$

A linear model is $y = 12{,}866{,}020 + 834{,}694.9x$.

The growth was 834,694.9 people per year.

b. Using the exponential regression feature on a graphing calculator, $y = 12{,}866{,}020(1.027)^x$.

The population growth was about 2.7% each year.

c. Linear model at $x = 20$:

$12{,}866{,}020 + 834{,}694.9(20) = 29{,}559{,}918$

Exponential model at $x = 20$:

$12{,}866{,}020(1.027)^{20} = 21{,}920{,}633.15$

Linear model at $x = 40$:

$12{,}866{,}020 + 834{,}694.9(40) = 46{,}253{,}816$

Exponential model at $x = 40$:

$12{,}866{,}020(1.027)^{40} = 37{,}347{,}536.99$

The exponential model is a better approximation of actual U.S. population for the time period 1850-1890.

47. $A = P\left(1 + \dfrac{r}{n}\right)^{nt} = 1000\left(1 + \dfrac{0.03}{1}\right)^{1(8)} = 1000(1.03)^8$

$= \$1266.77$

48. $A = P\left(1 + \dfrac{r}{n}\right)^{nt} = 1000\left(1 + \dfrac{0.03}{4}\right)^{4(8)} = 1000(1.0075)^{32}$

$= \$1270.11$

49. $A = P\left(1 + \dfrac{r}{n}\right)^{nt} = 1000\left(1 + \dfrac{0.03}{365}\right)^{365(8)}$

$= 1000(1.000082192)^{2920}$

$= \$1271.24$

50. Daily; in an account that is compounded daily, each day you earn interest on both the principal and the interest that was accrued on the previous days.

51. $A = P\left(1 + \dfrac{r}{n}\right)^{nt} = 500\left(1 + \dfrac{r}{12}\right)^{12(4)} = 500\left(1 + \dfrac{r}{12}\right)^{48}$

r	Amount
0.04 = 4%	\$586.60
0.041 = 4.1%	\$588.94
0.042 = 4.2%	\$591.29
0.043 = 4.3%	\$593.66
0.044 = 4.4%	\$596.03
0.045 = 4.5%	\$598.41
0.046 = 4.6%	\$600.80

The least annual interest rate is 4.6%.

Mixed Review

52. $\left(\dfrac{1}{3}\right)^2 = \dfrac{1^2}{3^2} = \dfrac{1}{9}$

53. $\left(\dfrac{1}{8}\right)^2 = \dfrac{1^2}{8^2} = \dfrac{1}{64}$

54. $\left(\dfrac{1}{4}\right)^3 = \dfrac{1^3}{4^3} = \dfrac{1}{64}$

55. $\left(\dfrac{1}{2}\right)^6 = \dfrac{1^6}{2^6} = \dfrac{1}{64}$

56. $\left(\dfrac{2}{3}\right)^{-2} = \left(\dfrac{3}{2}\right)^2 = \dfrac{3^2}{2^2} = \dfrac{9}{4}$

57. $\left(\dfrac{7}{5}\right)^{-2} = \left(\dfrac{5}{7}\right)^2 = \dfrac{5^2}{7^2} = \dfrac{25}{49}$

58. $\left(\dfrac{4}{3}\right)^{-3} = \left(\dfrac{3}{4}\right)^3 = \dfrac{3^3}{4^3} = \dfrac{27}{64}$

59. $\left(\dfrac{3}{2}\right)^{-4} = \left(\dfrac{2}{3}\right)^4 = \dfrac{2^4}{3^4} = \dfrac{16}{81}$

60. slope $= \dfrac{1 - (-3)}{5 - 0} = \dfrac{4}{5}$

$y = \dfrac{4}{5}x - 3$

61. slope $= \dfrac{3 - 0}{-1 - (-4)} = 1$

$y - 0 = 1(x + 4)$

$y = x + 4$

Problem Solving Workshop 8.5 (p. 529)

1. **a.** $b = 20(1 + 0.12)^t$

$b = 20(1.12)^t$

b. $b = 20(1.12)^{(1)} = \$22.40$

c. 2000; Using the fill down feature of the spreadsheet, you can see that when $t = 3$, $b = 28.1$. So the intercity bus fare was \$28.10 in 2000.

2. The error is that the growth rate is written in place of the growth factor. The function should be $b = 20(1.12)^t$.

3. **a.** $T = 7.5(1 + 0.039)^t$

$T = 7.5(1.039)^t$

b.

Months since May, 1997, *t*	Number of transistors, *T* (millions)
0	7.5
1	7.7925
2	8.0964
. . .	
40	34.648
41	36
42	37.404

About 37.4 million transistors in a CPU were released by the company in November 2000.

4. **a.** $V = 150{,}000(1 + 0.065)^t$

$V = 150{,}000(1.065)^t$

b.

Years since 2002, *t*	Value, *V* (dollars)
0	150,000
1	159,750
. . .	
4	192,970
5	205,513
6	218,871

When $t = 5$, or in 2007, the value of the home was about \$200,000.

Chapter 8, *continued*

Lesson 8.6

Investigating Algebra Activity 8.6 (p. 530)

Stage	**Number of pieces**	**Length of each new piece**
1	2	$\frac{1}{2}$
2	4	$\frac{1}{4}$
3	8	$\frac{1}{8}$
4	16	$\frac{1}{16}$
5	32	$\frac{1}{32}$

1. a. Yes; For each increase of 1 in the stage, the number of pieces is multiplied by 2.

b. $y = 2^x$

c. $y = 2^{10}$ there are 1024 pieces of yarn at stage 10.

2. a. Yes; For each increase of 1 in the stage; the length of each piece is multiplied by $\frac{1}{2}$.

b. $y = \left(\frac{1}{2}\right)^x$

c. $y = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$ units

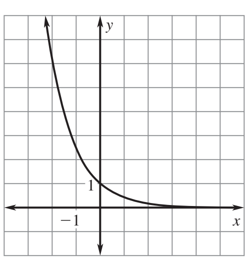
The length of each new piece of yarn at stage is $\frac{1}{1024}$ units

8.6 Guided Practice (pp. 531-534)

1. Yes; The y-values are multipied by $\frac{1}{5}$ for each increase of 1 in x, so the table represents an exponential function of the form $y = a \cdot \left(\frac{1}{5}\right)^x$. When $x = 0$, $y = 1 = a$. So, $y = \left(\frac{1}{5}\right)^x$.

2. $y = (0.4)^x$

x	−2	−1	0	1	2
y	6.25	2.5	1	0.4	0.16

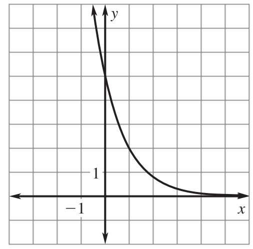


Domain: all real numbers

Range: all positive real numbers

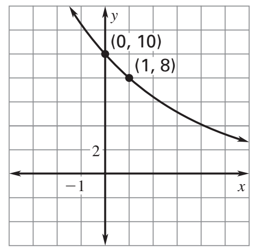
3. $y = 5 \cdot (0.4)^x$

x	−2	−1	0	1	2
y	31.25	12.5	5	2	0.8



The graph of $y = 5 \cdot (0.4)^x$ is a vertical stretch of the graph of $y = (0.4)^x$.

4.



The graph represents exponential decay. The y-intercept is 10, so $a = 10$,

$y = ab^x$

$8 = 10 \cdot b^1$

$0.8 = b$

A function rule is $y = 10(0.8)^x$.

5. $P = 41(0.995)^t = 41(0.995)^{47} \approx 32.4$

There will be about 32.4 million acres left in 2010.

8.6 Exercises (pp. 535–538)

Skill Practice

1. The decay factor in the exponential decay model $y = a(1 - r)^t$ is $1 - r$.

2. *Sample answer:* The graph of an exponential decay function falls from left to right while the graph of an exponential growth function rises from left to right.

3. The y-values are multiplied by 4 for each increase of 1 in x, so the table represents an exponential function of the form $y = a \cdot 4^x$, when $x = 0$, $y = 8 = a$. So, $y = 8 \cdot 4^x$.

4. The y-values are multiplied by $\frac{1}{5}$ for each increase of 1 in x, so the table represents an exponential function of the form $y = a \cdot \left(\frac{1}{5}\right)^x$. When $x = 0$, $y = 10 = a$.

So, $y = 10 \cdot \left(\frac{1}{5}\right)^x$.

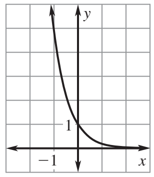
5. The y-values are multiplied by $\frac{1}{3}$ for each increase of 1 in x, so the table represents an exponential function of the form $y = a \cdot \left(\frac{1}{3}\right)^x$. When $x = 0$, $y = 2 = a$.

So, $y = 2 \cdot \frac{1}{3^x}$.

6. The y-values are increased by 4 for each increase of 1 in x, so the function is not exponential.

7. $y = \left(\frac{1}{5}\right)^x$

x	−1	0	1	2	3
y	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$

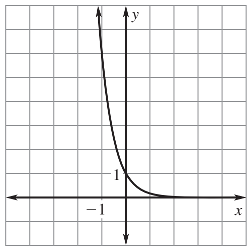


Domain: All real numbers

Range: $y > 0$

8. $y = \left(\frac{1}{6}\right)^x$

x	−1	0	1	2	3
y	6	1	$\frac{1}{6}$	$\frac{1}{36}$	$\frac{1}{216}$

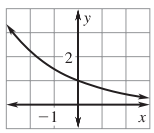


Domain: All real numbers

Range: $y > 0$

9. $y = \left(\frac{2}{3}\right)^x$

x	−2	−1	0	1	2
y	$\frac{9}{4}$	$\frac{3}{2}$	1	$\frac{2}{3}$	$\frac{4}{9}$

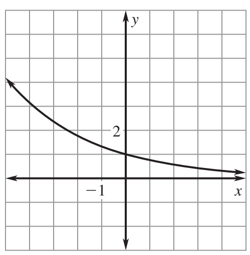


Domain: All real numbers

Range: $y > 0$

10. $y = \left(\frac{3}{4}\right)^x$

x	−3	−2	−1	0	1
y	$\frac{64}{27}$	$\frac{16}{9}$	$\frac{4}{3}$	1	$\frac{3}{4}$

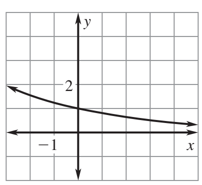


Domain: All real numbers

Range: $y > 0$

11. $y = \left(\frac{4}{5}\right)^x$

x	−2	−1	0	1	2
y	$\frac{25}{16}$	$\frac{5}{4}$	1	$\frac{4}{5}$	$\frac{16}{25}$

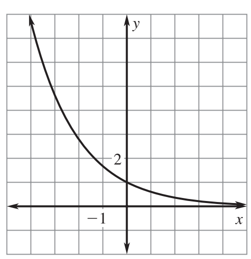


Domain: All real numbers

Range: $y > 0$

12. $y = \left(\frac{3}{5}\right)^x$

x	−4	−2	0	2	4
y	$\frac{625}{81}$	$\frac{25}{9}$	1	$\frac{9}{25}$	$\frac{81}{625}$

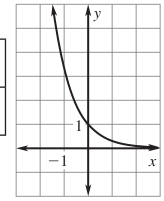


Domain: All real numbers

Range: $y > 0$

13. $y = (0.3)^x$

x	−2	−1	0	1	2
y	11.11	3.33	1	0.3	0.09

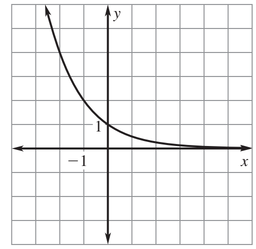


Domain: All real numbers

Range: $y > 0$

14. $y = (0.5)^x$

x	−2	−1	0	1	2
y	4	2	1	0.5	0.25

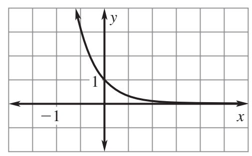


Domain: All real numbers

Range: $y > 0$

15. $y = (0.1)^x$

x	−1	0	1	2	3
y	10	1	0.1	0.01	0.001

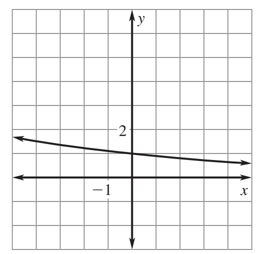


Domain: All real numbers

Range: $y > 0$

16. $y = (0.9)^x$

x	−6	−4	−2	0	2
y	1.9	1.5	1.2	1	0.8

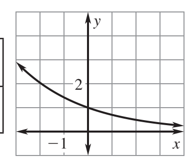


Domain: All real numbers

Range: $y > 0$

17. $y = (0.7)^x$

x	−4	−2	0	2	4
y	4.16	2.04	1	0.49	0.24

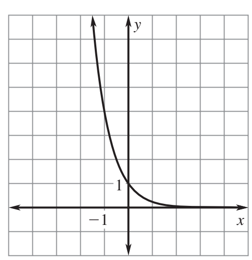


Domain: All real numbers

Range: $y > 0$

18. $y = (0.25)^x$

x	−2	−1	0	1	2
y	16	4	1	0.25	0.0625



Domain: All real numbers

Range: $y > 0$

19. D; When $x = 0$, $y = 4 = a$.

$y = 4 \cdot b^x$

$2 = 4 \cdot b^1$

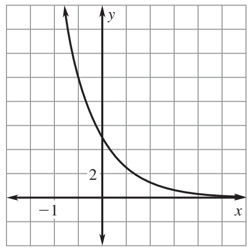
$2 = 4b$

$0.5 = b$

The function is $y = 4 \cdot (0.5)^x$.

20. $y = 5 \cdot \left(\frac{1}{4}\right)^x$

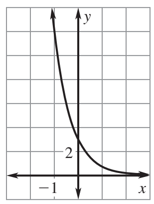
x	−2	−1	0	1	2
y	80	20	5	$\frac{5}{4}$	$\frac{5}{16}$



The graph of $y = 5 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch of the graph of $y = \left(\frac{1}{4}\right)^x$.

21. $y = 3 \cdot \left(\frac{1}{4}\right)^x$

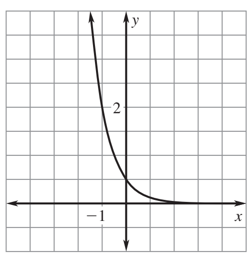
x	−2	−1	0	1	2
y	48	12	3	$\frac{3}{4}$	$\frac{3}{16}$



The graph of $y = 3 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch of the graph of $y = \left(\frac{1}{4}\right)^x$.

22. $y = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$

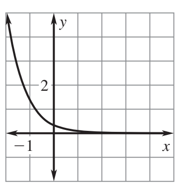
x	−2	−1	0	1	2
y	8	2	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{32}$



The graph of $y = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink of the graph of $y = \left(\frac{1}{4}\right)^x$.

23. $y = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$

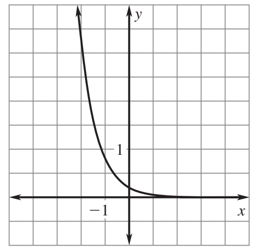
x	−2	−1	0	1	2
y	$\frac{16}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{48}$



The graph of $y = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink of the graph of $y = \left(\frac{1}{4}\right)^x$.

24. $y = 0.2 \cdot \left(\frac{1}{4}\right)^x$

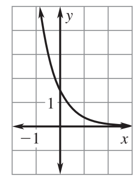
x	−2	−1	0	1	2
y	3.2	0.8	0.2	0.05	0.01



The graph of $y = 0.2 \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink of the graph of $y = \left(\frac{1}{4}\right)^x$.

25. $y = 1.5 \cdot \left(\frac{1}{4}\right)^x$

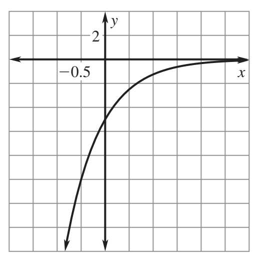
x	−2	−1	0	1	2
y	24	6	$\frac{3}{2}$	$\frac{3}{8}$	$\frac{3}{32}$



The graph of $y = 1.5 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch of the graph of $y = \left(\frac{1}{4}\right)^x$.

26. $y = -5 \cdot \left(\frac{1}{4}\right)^x$

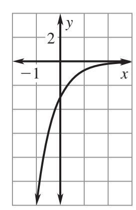
x	−2	−1	0	1	2
y	−80	−20	−5	$-\frac{5}{4}$	$-\frac{5}{16}$



The graph of $y = -5 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch with a reflection in the x-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

27. $y = -3 \cdot \left(\frac{1}{4}\right)^x$

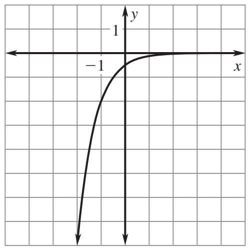
x	−2	−1	0	1	2
y	−48	−12	−3	$-\frac{3}{4}$	$-\frac{3}{16}$



The graph of $y = -3 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch with a reflection in the x-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

28. $y = -\frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$

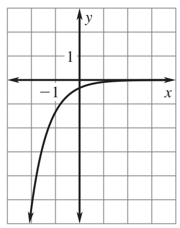
x	−2	−1	0	1	2
y	−8	−2	$-\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{32}$



The graph of $y = -\frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink with a reflection in the x-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

29. $y = -\frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$

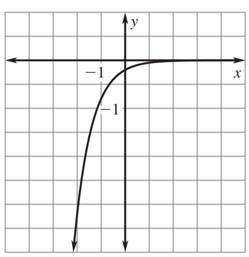
x	−2	−1	0	1	2
y	$-\frac{16}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{12}$	$-\frac{1}{48}$



The graph of $y = -\frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink with a reflection in the x-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

30. $y = -0.2 \cdot \left(\frac{1}{4}\right)^x$

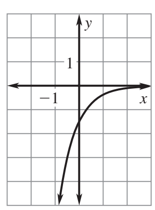
x	−2	−1	0	1	2
y	−3.2	−0.8	−0.2	−0.05	−0.01



The graph of $y = -0.2 \cdot \left(\frac{1}{4}\right)^x$ is a vertical shrink with a reflection in the x-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

31. $y = -1.5 \cdot \left(\frac{1}{4}\right)^x$

x	−2	−1	0	1	2
y	−24	−6	$-\frac{3}{2}$	$-\frac{3}{8}$	$-\frac{3}{32}$



The graph of $y = -1.5 \cdot \left(\frac{1}{4}\right)^x$ is a vertical stretch with a reflection in the x-axis of the graph of $y = \left(\frac{1}{4}\right)^x$.

32. A; $y = (0.2)^x$

Let $x = 0$: $y = (0.2)^0 = 1$

y-intercept: (0, 1)

33. C; $y = 5(0.2)^x$

Let $x = 0$: $y = 5(0.2)^0 = 5$

y-intercept: (0, 5)

34. B; $y = \frac{1}{2}(0.2)^x$

Let $x = 0$: $y = \frac{1}{2}(0.2)^0 = \frac{1}{2}$

y-intercept: $\left(0, \frac{1}{2}\right)$

35. $y = a(1 - r)^t$

initial amount $= a = 90{,}000$

decay rate $= r = 0.025$

decay factor $= 1 - r = 1 - 0.025 = 0.975$

$y = 90{,}000(0.975)^t$

36. D;

decay factor $= 1 -$ decay rate

$0.97 = 1 - r$

$r = 1 - 0.97$

$r = 0.03$

37. The error is that the decay rate was placed where the decay factor should be. The equation should be:

$y = a(1 - r)^t = 25{,}000(1 - 0.14)^t = 25{,}000(0.86)^t$

38. The graph represents exponential decay because it falls from left to right. The y-intercept is 6, so $a = 6$.

$y = ab^x$

$4.8 = 6b^1$

$0.8 = b$

The function rule is $y = 6(0.8)^x$.

39. The graph represents exponential decay because it falls from left to right. The y-intercept is 8, so $a = 8$.

$y = ab^x$

$4.8 = 8b^1$

$0.6 = b$

The function rule is $y = 8(0.6)^x$.

40. The graph represents exponential growth because it rises from left to right. The y-intercept is 8, so $a = 8$.

$y = ab^x$

$12.8 = 8b^1$

$1.6 = b$

The function rule is $y = 8(1.6)^x$.

41. a. $m(x)$ is a vertical shrink of $f(x)$.

b. $n(x)$ is a vertical stretch with a reflection in the x-axis of $f(x)$.

c. $p(x)$ is a vertical translation of 1 unit up of $f(x)$.

42. (0, 1), $\left(2, \frac{1}{4}\right)$

y is multiplied by $\frac{1}{2}$ for each increase of 1 in x, so $b = \frac{1}{2}$.

(When $x = 0$, $y = 1 = a$. A function rule is $y = \left(\frac{1}{2}\right)^x$.

43. (1, 20), (2, 4)

y is multiplied by $\frac{1}{5}$ for each increase of 1 in x, so $b = \frac{1}{5}$.

$y = a\left(\frac{1}{5}\right)^x$

$20 = a\left(\frac{1}{5}\right)^1$

$100 = a$

A function rule is $y = 100\left(\frac{1}{5}\right)^x$.

44. $\left(1, \frac{3}{2}\right), \left(2, \frac{3}{4}\right)$

y is multiplied by $\frac{1}{2}$ for each increase of 1 in x, so $b = \frac{1}{2}$.

$y = a\left(\frac{1}{2}\right)^x$

$\frac{3}{2} = a\left(\frac{1}{2}\right)^1$

$3 = a$

A function rule is $y = 3\left(\frac{1}{2}\right)^x$.

45. Since the quantity loses the same percent each time period, the amount y remaining after t time periods can be modeled using the exponential function $y = a(1 - r)^t$. If it is after one time period, $t = 1$.

$y = a(1 - r)^t = a(1 - r)^1 = a(1 - r)$

By replacing t with 1 and using exponential properties, it can be shown that after one time period the amount of the quantity equals $a(1 - r)$.

46. The graphs are the same graph.

$f(x) = 4^{x-2}$

$= 4^x \cdot 4^{(-2)}$

$= 4^x \cdot \frac{1}{16}$

$= \frac{1}{16} \cdot 4^x = g(x)$

Problem Solving

47. Let y represent the value of the cell phone and t represent the number of years since purchase,

$y = 125(1 - 0.2)^t = 125(0.8)^t = 125(0.8)^3 = 64$

The value of the cell phone after 3 years is \$64.

48. a. Initial amount $= a = 141{,}200$

Decay rate $= r = 0.11$

Decay Factor $= 1 - r = 1 - 0.11 = 0.89$

b. Let B represent the number of bats and t represent the number of years since 1983.

$y = 141{,}200(0.89)^t = 141{,}200(0.89)^{20} \approx 13{,}729$

There were about 13,729 bats in 2003.

49. No; in 2006, the boat will be worth

$B = 4000(0.93)^3 = \$3217.43$.

Selling the boat for \$3000 will be selling the boat for less than what it's worth.

50. a. Initial amount $= 128$

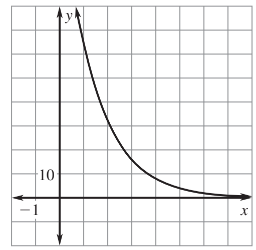
Decay factor $= \frac{1}{2}$

$y = 128\left(\frac{1}{2}\right)^x$

b.

Rounds completed	0	1	2	3	4	5	6	7
Teams remaining	128	64	32	16	8	4	2	1

c.



After round 5 there will be 4 teams left in the tournament.

51. a. Decay factor $= 0.9439$

Decay Rate $= 1 - 0.9439 = 0.0561$

b. $d = 1.516(0.9439)^1 \approx 1.431$ inches

c. The distance between the nut and the first fret is

$d = \frac{1}{2}\left[1.516(0.9439)^1\right] = \frac{1}{2}(1.431) \approx 0.716$ inches

The distance between the 12th and 13th frets is

$d = 1.516(0.9439)^{13} \approx 0.716$ inches.

52. Let y represent the remaining balance and t represent the number of months since purchase.

$y = 1850(1 - 0.0225)^t = 1850(0.9775)^t$

After the 23rd month, the remaining balance is

$y = 1850(0.9775)^{23} = 1096.12$.

If the student buys the computer without paying interest, he should pay the remaining balance \$1096.12 after the 23rd month.

53. a. 1st athlete

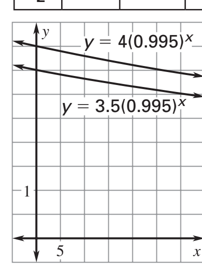
$y_1 = 4(1 - 0.005)^x = 4(0.995)^x$

2nd athlete

$y_2 = 3.5(1 - 0.005)^x = 3.5(0.995)^x$

b.

x	0	10	20	30	40	50	60
y_1	4	3.80	3.62	3.44	3.27	3.11	2.96
y_2	3.5	3.33	3.17	3.01	2.86	2.72	2.59



c. At about $x = 26$, $y \approx 3.5$. So the first athlete will be about $25 + 26 = 51$ years old when her maximal oxygen consumption is equal to 3.5 liters per minute.

Mixed Review

54. $-12x + (-3x) = (-12 - 3)x = -15x$

55. $8x - 3x = (8 - 3)x = 5x$

56. $14 + x + 2x = 14 + (1 + 2)x = 14 + 3x$

57. $7(2x + 1) - 5 = 7(2x) + 7(1) - 5$
$= 14x + 7 - 5$
$= 14x + 2$

58. $13x + (x - 4)5 = 13x + x(5) - 4(5)$
$= 13x + 5x - 20$
$= (13 + 5)x - 20$
$= 18x - 20$

59. $3x + 6(x + 9) = 3x + 6(x) + 6(9)$
$= 3x + 6x + 54$
$= (3 + 6)x + 54$
$= 9x + 54$

60. $(5 - x) + x = 5 + (-x + x) = 5$

61. $(3x - 4)7 + 21 = 3x(7) - 4(7) + 21$
$= 21x - 28 + 21$
$= 21x - 7$

62. $-(x - 1) - x^2 = -x + 1 - x^2 = 1 - x - x^2$

63. $x + 14 = 8$
$x = 8 - 14$
$x = -6$

64. $8x - 7 = 17$
$8x = 17 + 7$
$8x = 24$
$x = \frac{24}{8}$
$x = 3$

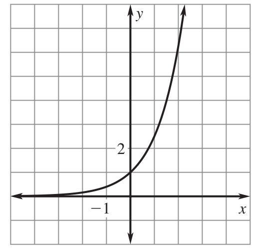
65. $4x + 2x - 6 = 18$
$(4 + 2)x - 6 = 18$
$6x - 6 = 18$
$6x = 18 + 6$
$6x = 24$
$x = \frac{24}{6}$
$x = 4$

66. $2x - 7(x + 5) = 20$
$2x - 7x - 7(5) = 20$
$(2 - 7)x - 35 = 20$
$-5x - 35 = 20$
$-5x = 20 + 35$
$-5x = 55$
$x = -11$

Quiz 8.5–8.6 (p. 538)

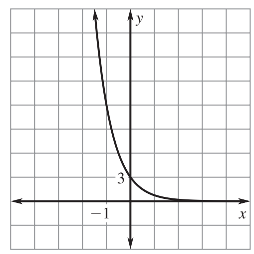
1. $y = \left(\frac{5}{2}\right)^x$

x	-2	-1	0	1	2
y	$\frac{4}{25}$	$\frac{2}{5}$	1	$\frac{5}{2}$	$\frac{25}{4}$



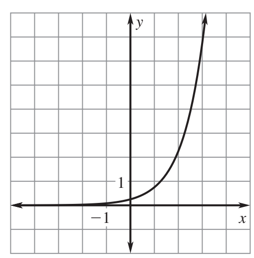
2. $y = 3 \cdot \left(\frac{1}{4}\right)^x$

x	-2	-1	0	1	2
y	48	12	3	$\frac{3}{4}$	$\frac{3}{16}$



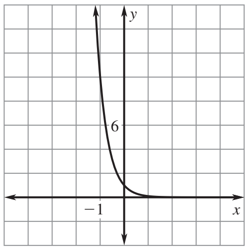
3. $y = \frac{1}{4} \cdot 3^x$

x	-1	0	1	2	3
y	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{9}{4}$	$\frac{27}{4}$



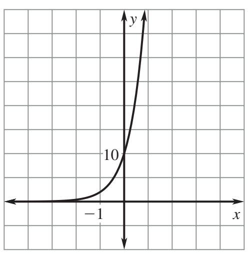
4. $y = (0.1)^x$

x	−1	0	1	2	3
y	10	1	0.1	0.01	0.001



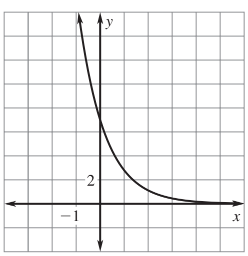
5. $y = 10 \cdot 5^x$

x	−2	−1	0	1	2
y	0.4	2	10	50	250



6. $y = 7(0.4)^x$

x	−2	−1	0	1	2
y	43.75	17.5	7	2.8	1.12



7. Let y represent the value of the coin and t represent the number of years since purchase.

$y = 25(1 + 0.08)^t = 25(1.08)^t$

When $t = 10$: $y = 25(1.08)^{10} = 53.97$

The value of the coin after 10 years is $53.97.

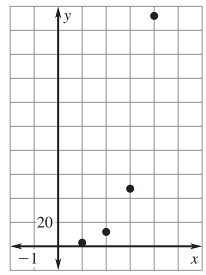
8.6 Extension (p. 540)

1. The ratios of consecutive terms are:

$\frac{a_2}{a_1} = \frac{12}{3} = 4, \frac{a_3}{a_2} = \frac{48}{12} = 4, \frac{a_4}{a_3} = \frac{192}{48} = 4$

The common ratio is 4, so the sequence is geometric.

Position, x	1	2	3	4
Term, y	3	12	48	192



2. The difference of consecutive terms are:

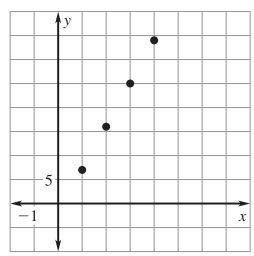
$a_2 - a_1 = 16 - 7 = 9$

$a_3 - a_2 = 25 - 16 = 9$

$a_4 - a_3 = 34 - 25 = 9$

The common difference is 9, so the sequence is arithmetic.

Position, x	1	2	3	4
Term, y	7	16	25	34



3. The difference of consecutive terms are:

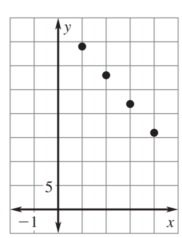
$a_2 - a_1 = 28 - 34 = -6$

$a_3 - a_2 = 22 - 28 = -6$

$a_4 - a_3 = 16 - 22 = -6$

The common difference is −6, so the sequence is arithmetic.

Position, x	1	2	3	4
Term, y	34	28	22	16

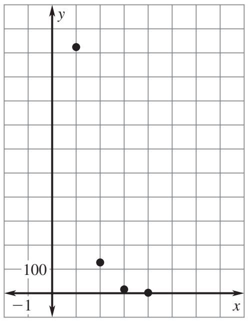


4. The ratios of consecutive terms are:

$\frac{a_2}{a_1} = \frac{128}{1024} = \frac{1}{8}, \frac{a_3}{a_2} = \frac{16}{128} = \frac{1}{8}, \frac{a_4}{a_3} = \frac{2}{16} = \frac{1}{8}$

The common ratio is $\frac{1}{8}$, so the sequence is geometric.

Position, *x*	1	2	3	4
Term, *y*	1024	128	16	2

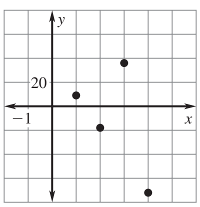


5. The ratios between consecutive terms are:

$\frac{a_2}{a_1} = \frac{-18}{9} = -2, \frac{a_3}{a_2} = \frac{36}{-18} = -2, \frac{a_4}{a_3} = \frac{-72}{36} = -2$

The common ratio is -2, so the sequence is geometric.

Position, *x*	1	2	3	4
Term, *y*	9	−18	36	−72



6. The differences between consecutive terms are:

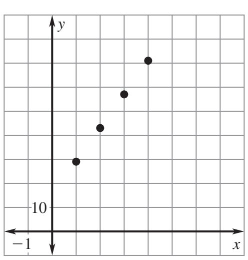
$a_2 - a_1 = 43 - 29 = 14$

$a_3 - a_2 = 57 - 43 = 14$

$a_4 - a_3 = 71 - 57 = 14$

The common difference is 14, so the sequence is arithmetic.

Position, *x*	1	2	3	4
Term, *y*	29	43	57	71



7. $r = \frac{a_2}{a_1} = \frac{-5}{1} = -5$

$a_n = (-5)^{n-1}$

$a_7 = (-5)^{7-1} = 15{,}625$

8. $r = \frac{a_2}{a_1} = \frac{26}{13} = 2$

$a_n = 13 \cdot 2^{n-1}$

$a_7 = 13 \cdot 2^{7-1} = 832$

9. $r = \frac{a_2}{a_1} = \frac{72}{432} = \frac{1}{6}$

$a_n = 432 \cdot \left(\frac{1}{6}\right)^{n-1}$

$a_7 = 432 \cdot \left(\frac{1}{6}\right)^{7-1} = \frac{1}{108}$

10. $\frac{a_2}{a_1} = \frac{4}{1} = 4$

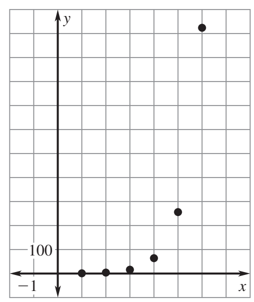
$\frac{a_3}{a_2} = \frac{16}{4} = 4$

$\frac{a_4}{a_3} = \frac{64}{16} = 4$

The ratios of consecutive terms are constant so the series is geometric where $r = 4$.

$a_n = 1 \cdot 4^{n-1} = 4^{n-1}$

Position, *x*	1	2	3	4	5	6
Term, *y*	1	4	16	64	256	1024



Mixed Review of Problem Solving (p. 541)

1. a. Sun: $96{,}600{,}000 = 9.66 \times 10^7$ km

Earth: $6370 = 6.37 \times 10^3$ km

b. Sun: $S = 4\pi r^2$

$= 4\pi(9.66 \times 10^7)^2$

$= (4\pi \cdot 9.66^2) \times (10^7)^2$

$\approx 1172 \cdot 64 \times 10^{14}$

$= (1.17264 \times 10^3) \times 10^{14}$

$= 1.17264 \times 10^{17}$

Earth: $S = 4\pi r^2$

$= 4\pi(6.37 \times 10^3)^2$

$= (4\pi \cdot 6.37^2) \times (10^3)^2$

$= 509.9 \times 10^6$

$= (5.099 \times 10^2) \times 10^6$

$= 5.099 \times 10^8$

c. $\frac{\text{Sun}}{\text{Earth}} = \frac{1.17264 \times 10^{17}}{5.099 \times 10^{8}}$

$= \frac{1.17264}{3.099} \times \frac{10^{17}}{10^{8}}$

$\approx 0.23 \times 10^{9}$

$= (2.3 \times 10^{-1}) \times 10^{9}$

$= 2.3 \times 10^{8}$

The surface area of the sun is about 2.3×10^{8} times larger than the surface area of Earth.

2. a. The y-intercept is 25,000, so $a = 25{,}000$.

$y = ab^x$

$23{,}750 = 25{,}000b^1$

$0.95 = b$

A function rule is $y = 25{,}000(0.95)^x$.

b. The decay factor, $1 - r$, for the truck is 0.95. So,

$1 - r = 0.95$

$0.05 = r$

The decay rate is 0.05, or 5%.

3. Decay factor $= 1 - r$

$0.82 = 1 - r$

$r = 0.18$

4. *Sample answer:* Let y represent the value of the house and t represent the number of quarters since 2001.

$y = a(1 + 0.04)^t = a(1.04)^t$

When $t = 4$,

$275{,}000 = a(1.04)^4$

$275{,}000 = 1.1699a$

$235{,}071.15 = a$

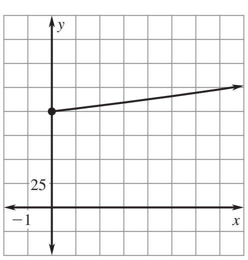
The value of the house can be modeled by $y = 235{,}000(1.04)^t$. A house with a value of \$235,000 at the end of 2001 would have a value of about \$275,000 at the end of 2002.

5. a. Let y represent the amount of money in the account and t represent the number of years since the \$100 was deposited,

$y = a(1 + r)^t = 100(1 + 0.03)t = 100(1.03)t$

b.

x	0	1	2	3	4
y	100	103	106.09	109.27	112.55



c. No, after 3 years the musician will only have \$109.27.

6. a. The graph rises from left to right, so it represents exponential growth.

b. The y-intercept is 15,000, so $a = 15{,}000$.

$y = ab^x$

$19{,}500 = 15{,}000b^1$

$1.3 = b$

A function rule is $y = 15{,}000(1.3)^x$.

c. $y = 15{,}000(1.3)^x = 15{,}000(1.3)^4 = 42{,}841.50$

The business is worth \$42,841.50 after 4 years.

7. a. Let y represent the amount of medication in patient's bloodstream (in milligrams) and let t represent the time since the medication was taken. The amount of medication is halved every 8 hours, so

$y = 500(0.5)^{t/8}$.

b. $y = 500(0.5)^{24/8} = 500(0.5)^3 = 62.5$

After 24 hours, the patient's bloodstream will have 62.5 mg of the medication.

Chapter 8 Review (pp. 543–546)

1. The function $y = 1200(0.3)^t$ is an exponential *decay* function, and the base 0.3 is called the *decay factor*.

2. *Sample answer:* A table represents a linear function if the output values change by the addition of the same number. A table represents an exponential function if the output values change by the multiplication of the same number.

3. The function $y = 3(0.85)^x$ represents exponential decay because the base, 0.85, is less than 1.

4. The function $y = \frac{1}{2}(1.01)^x$ represents exponential growth because the base, 1.01, is greater than 1.

5. The function $y = 2(2.1)^x$ represents exponential growth because the base, 2.1, is greater than 1.

6. $4^4 \cdot 4^3 = 4^{4+3} = 4^7$

7. $(-3)^7(-3) = (-3)^{7+1} = (-3)^8$

8. $z^3 \cdot z^5 \cdot z^5 = z^{3+5+5} = z^{13}$

9. $(y^4)^5 = y^{4 \cdot 5} = y^{20}$

10. $[(-7)^4]^4 = (-7)^{4 \cdot 4} = (-7)^{16}$

11. $[(b + 2)^8]^3 = (b + 2)^{8 \cdot 3} = (b + 2)^{24}$

12. $(6^4 \cdot 31)^5 = 6^{4 \cdot 5} \cdot 31^5 = 6^{20} \cdot 31^5$

13. $-(8xy)^2 = -8^2x^2y^2 = -64x^2y^2$

14. $(2x^2)^4 \cdot x^5 = 2^4x^{2 \cdot 4}x^5 = 16x^8x^5 = 16x^{8+5} = 16x^{13}$

15. $10^{18} \cdot 10^3 = 10^{18+3} = 10^{21}$ kilograms

16. $\frac{(-3)^7}{(-3)^3} = (-3)^{7-3} = (-3)^4$

17. $\frac{5^2 \cdot 5^4}{5^3} = \frac{5^{2+4}}{5^3} = \frac{5^6}{5^3} = 5^{6-3} = 5^3$

18. $\left(\frac{m}{n}\right)^3 = \frac{m^3}{n^3}$

19. $\frac{17^{12}}{17^8} = 17^{12-8} = 17^4$

20. $\left(-\frac{1}{x}\right)^4 = \frac{(-1)^4}{x^4}$

$= \frac{1}{x^4}$

21. $\left(\frac{7x^5}{y^2}\right)^2 = \frac{7^2x^{5 \cdot 2}}{y^{2 \cdot 2}}$

$= \frac{49x^{10}}{y^4}$

22. $\frac{1}{p^2} \cdot p^6 = p^{6-2} = p^4$

23. $\frac{6}{7r^{10}} \cdot \left(\frac{r^5}{s}\right)^5 = \frac{6}{7r^{10}} \cdot \frac{r^{5 \cdot 5}}{s^5} = \frac{6r^{25}}{7r^{10}s^5} = \frac{6r^{25-10}}{7s^5} = \frac{6r^{15}}{7s^5}$

24. $\frac{10^{10}}{10^6} = 10^{10-6} = 10^4$

The order of magnitude of the mean personal income in Montana in 2003 was $\$10^4$.

25. $14^0 = 1$

26. $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

27. $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$

28. $7^{-5} \cdot 7^5 = 7^{-5+5} = 7^0 = 1$

29. $\frac{1 \text{ femtogram}}{10^{-18} \text{ kilogram}} \cdot \frac{10^{-12} \text{ kilogram}}{1 \text{ nanogram}} = 10^{-12-(-18)}$

$= \frac{10^6 \text{ femtogram}}{1 \text{ nanogram}}$

30. $78{,}120 = 7{,}812 \times 10^4$

31. $7.5 \times 10^{-5} = 0.000075$

32. $(6.3 \times 10^3)(1.9 \times 10^{-5}) = (6.3 \cdot 1.9) \times (10^3 \cdot 10^{-5})$

$= 11.97 \times 10^{-2}$

$= 1.197 \times 10^1 \times 10^{-2}$

$= 1.197 \times 10^{-1}$

33. $\frac{6.5 \times 10^9}{1.6 \times 10^{-4}} = \frac{6.5}{1.6} \times \frac{10^9}{10^{-4}} = 4.0625 \times 10^{13}$

34. $\frac{m_1}{m_2} = \frac{1.5 \times 10^6}{6 \times 10^9}$

$= \frac{1.5}{6} \times \frac{10^6}{10^9}$

$= 0.25 \times 10^{-3}$

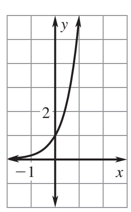
$= 2.5 \times 10^{-1} \times 10^{-3}$

$= 2.5 \times 10^{-4}$

The mass of the Great Pyramid of Giza is 25,000 times the mass of a gate of the Thames Barrier.

35. $y = 6^x$

x	−1	0	1	2
y	$\frac{1}{6}$	1	6	36

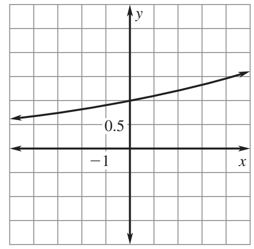


Domain: all real numbers

Range: all positive real numbers

36. $y = (1.1)^x$

x	−1	0	1	2
y	0.91	1	1.1	1.21

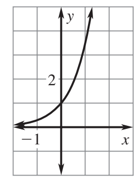


Domain: all real numbers

Range: all positive real numbers

37. $y = (3.5)^x$

x	−1	0	1	2
y	0.29	1	3.5	12.25

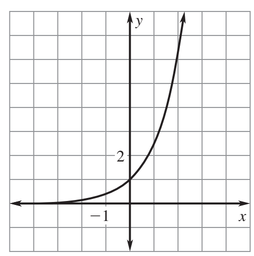


Domain: all real numbers

Range: all positive real numbers

38. $y = \left(\frac{5}{2}\right)^x$

x	−1	0	1	2
y	$\frac{2}{5}$	1	$\frac{5}{2}$	$\frac{25}{4}$

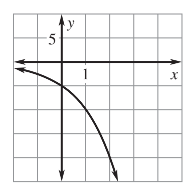


Domain: all real numbers

Range: all positive real numbers

39. $y = -5 \cdot 2^x$

x	−1	0	1	2
y	$-\frac{5}{2}$	−5	−10	−20



The graph of $y = -5 \cdot 2^x$ is a vertical stretch with a reflection in the x-axis of the graph of $y = 2^x$.

40. The graph represents exponential growth. The y-intercept is 1, so $a = 1$.

$y = ab^x$

$4 = 1 \cdot b^1$

$4 = b$

A function rule is $y = 4^x$.

41. The graph represents exponential decay. The y-intercept is 3, so $a = 3$.

$y = ab^x$

$1 = 3b^1$

$\frac{1}{3} = b$

A function rule is $y = 3\left(\frac{1}{3}\right)^x$.

42. Let V represent the value of the car (in dollars) and t represent the number of years since the car was purchased.

$V = a(1 - r)^t = 13{,}000(1 - 0.15)^t = 13{,}000(0.85)^t$

Subsitute 4 for t.

$V = 13{,}000(0.85)^t = 13{,}000(0.85)^4 \approx 6786.08$

The approximate value of the car in 4 years is \$6786.08.

Chapter 8 Test (p. 547)

1. $(62 \cdot 17)^4 = 62^4 \cdot 17^4$

2. $(-3)(-3)^6 = (-3)^{1+6} = (-3)^7$

3. $\frac{8^4 \cdot 8^5}{8^3} = \frac{8^{4+5}}{8^3} = 8^{9-3} = 8^6$

4. $(8^4)^3 = 8^{4 \cdot 3} = 8^{12}$ **5.** $\frac{2^{15}}{2^8} = 2^{15-8} = 2^7$

6. $5^3 \cdot 5^0 \cdot 5^5 = 5^{3+0+5} = 5^8$

7. $[(-4^3)]^2 = (-4)^{3 \cdot 2} = (-4)^6$

8. $\frac{(-5)^{10}}{(-5)^3} = (-5)^{10-3} = (-5)^7$

9. $t^2 \cdot t^6 = t^{2+6} = t^8$

10. $\left(\frac{s}{t}\right)^6 = \frac{s^6}{t^6}$ **11.** $\frac{1}{9^{-2}} = 9^2 = 81$

12. $-(6p)^2 = -6^2p^2 = -36p^2$

13. $(5xy)^2 = 5^2x^2y^2 = 25x^2y^2$

14. $\frac{1}{z^7} \cdot z^9 = z^{9-7} = z^2$

15. $(x^5)^3 = x^{5 \cdot 3} = x^{15}$

16. $\left(\frac{-4}{c}\right)^2 = \frac{(-4)^2}{c^2} = \frac{16}{c^2}$

17. $\left(\frac{a^{-3}}{3b}\right)^4 = \frac{a^{-3 \cdot 4}}{(3b)^4} = \frac{a^{-12}}{3^4b^4} = \frac{1}{81a^{12}b^4}$

18. $\frac{3}{4d} \cdot \frac{(2d)^4}{c^3} = \frac{3}{4d} \cdot \frac{2^4d^4}{c^3} = \frac{3 \cdot 16d^4}{4dc^3} = \frac{12d^{4-1}}{c^3} = \frac{12d^3}{c^3}$

19. $y^0 \cdot (8x^6y^{-3})^{-2} = 1 \cdot 8^{-2}x^{6(-2)}y^{(-3)(-2)}$

$= \frac{1}{64} \cdot x^{-12}y^6$

$= \frac{y^6}{64x^{12}}$

20. $(5r^5)^3 \cdot r^{-2} = 5^3r^{5 \cdot 3}r^{-2}$

$= 125r^{15}r^{-2}$

$= 125r^{15-2}$

$= 125r^{13}$

21. $423.6 = 4.236 \times 10^2$

22. $7{,}194{,}548 = 7.194548 \times 10^6$

23. $500.32 = 5.0032 \times 10^2$

24. $71.23884 = 7.123884 \times 10^1$

25. $0.562 = 5.62 \times 10^{-1}$ **26.** $0.0348 = 3.48 \times 10^{-2}$

27. $0.000123 = 1.23 \times 10^{-4}$

28. $0.5603002 = 5.603002 \times 10^{-1}$

29. $4.02 \times 10^5 = 402{,}000$ **30.** $5.3121 \times 10^4 = 53{,}121$

31. $9.354 \times 10^8 = 935{,}400{,}000$

32. $1.307 \times 10^{19} = 13{,}070{,}000{,}000{,}000{,}000{,}000$

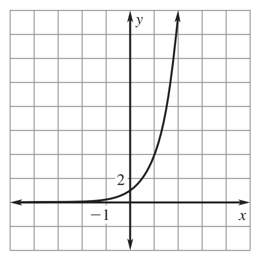
33. $1.3 \times 10^{-3} = 0.0013$ **34.** $3.32 \times 10^{-4} = 0.000332$

35. $7.506 \times 10^{-5} = 0.00007506$

36. $9.3119 \times 10^{-7} = 0.00000093119$

37. $y = 4^x$

x	-1	0	1	2
y	$\frac{1}{4}$	1	4	16

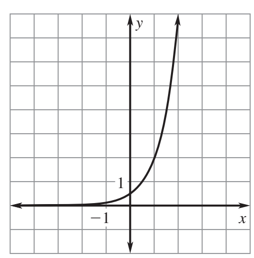


Domain: all real numbers

Range: all positive real numbers

38. $y = \frac{1}{2} \cdot 4^x$

x	-1	0	1	2
y	$\frac{1}{8}$	$\frac{1}{2}$	2	8



The graph of $y = \frac{1}{2}. 4^x$ is a vertical shrink of the graph of $y = 4^x$.

39. $\frac{1.2 \times 10^7 \text{ bytes}}{1 \text{ frame}} \cdot \frac{24 \text{ frames}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}$

$= (1.2 \cdot 24 \cdot 60 \cdot 60) \times 10^7$

$= 103{,}680 \times 10^7$

$\approx 1.04 \times 10^5 \times 10^7$

$= 1.04 \times 10^{12}$

There are about $1.0^4 \times 10^{12}$ bytes of data in 1 hour of an animated film.

40. a. Let y represent the yearly salary and t represent the number of years since accepting the job.

$y = 32{,}000(1 + 0.03)^t$

$y = 32{,}000(1.03)^t$

b. $y = 32{,}000(1.03)^t = 32{,}000(1.03)^5 = 37{,}096.77$

The employee's salary after 5 years is $37,096.77.

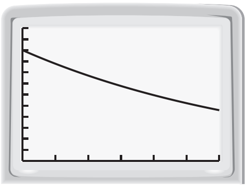
41. a. $P = (0.99987)^a$

Initial amount: 1

Decay factor: $1 - r = 0.99987$

Decay rate: $r = 0.00013$

b.



c. At $a = 5332$ meters, the atmospheric pressure is about 0.5 atmosphere.

Standardized Test Preparation (p. 549)

1. C; $V = \ell wh = 30 \cdot 12 \cdot 12 = 4320$

The order of magnitude of the volume of water is 10^4 cubic inches.

$10^4 \cdot 10^{-2} = 10^{4-2} = 10^2$

The order of magnitude of the weight of the water is 10^2 pounds.

2. A; $V = \pi r^2 h = \pi(3x)^2 x = \pi 3^2 x^2 x = 9\pi x^3$

Standardized Test Practice (pp. 550–551)

1. A; For each increase of 1 in x, y is multiplied by 5.

The y-intercept is $\frac{1}{3}$, so $a = \frac{1}{3}$.

$y = ab^x$

$\frac{5}{3} = \frac{1}{3}b^1$

$5 = b$

A function rule is $y = \frac{1}{3} \cdot 5^x$.

2. D; $V = s^3 = (4x)^3 = 4^3x^3 = 64x^3$

3. D

Elements in Seawater	Concentration (parts per million)
Sulfur	$904 = 9.04 \times 10^2$
Chloride	1.95×10^4
Magnesium	1.29×10^3
Sodium	$10.770 = 1.077 \times 10^4$

$9.04 \times 10^2 < 1.29 \times 10^3 < 1.077 \times 10^4 < 1.95 \times 10^4$

The order from least to greatest is sulfur, magnesium, sodium, chloride.

4. D; $\frac{\text{chloride}}{\text{magnesium}} = \frac{1.95 \times 10^4}{1.29 \times 10^3}$

$= \frac{1.95}{1.29} \times \frac{10^4}{10^3}$

$\approx 1.51 \times 10^1$

≈ 15

5. D; $\frac{1 \text{ mm}}{10^{-3} \text{ m}} \cdot \frac{10^3 \text{ m}}{1 \text{ km}} = 10^{3-(-3)} = \frac{10^6 \text{ mm}}{1 \text{ km}}$

6. C; $\frac{1 \text{ nm}}{10^{-9} \text{ m}} \cdot \frac{10^{-2} \text{ m}}{1 \text{ cm}} = 10^{-2-(-9)} = \frac{10^7 \text{ nm}}{1 \text{ cm}}$

7. C; $10^{-9} \times 10^3 = 10^{-9+3} = 10^{-6}$

8. D; The y-intercept is 2, so $a = 2$.

$y = ab^x$

$6 = 2b^1$

$3 = b$

A function rule is $y = 2 \cdot 3^x$.

9. A; The graph of $y = 2 \cdot 3^x$ is a vertical stretch of the graph of $y = 3^x$.

10. $\left(\frac{x^{-6}}{x^{-5}}\right)^{-3} = \frac{x^{-6(-3)}}{x^{-5(-3)}} = \frac{x^{18}}{x^{15}} = x^{18-15} = x^3$

The value of n is 3.

11. $7.8 \times 10^{-1} = 0.78$

12. $12{,}560{,}000 = 1.256 \times 10^7$

The power of 10 is 7.

13. For each increase of 1 in x, the value of y is multiplied by 5. The missing value is $15 \cdot 5 = 75$.

14. Let y represent the value of the investment and t represent the number of years since the initial investment.

$y = 200(1 - 0.015)^t = 200(0.985)^3 = 191.134$

After 3 years, the value is $191.13.

15. $(2x)^3 \cdot x^2 = 2^3x^3x^2$
$= 8x^{3+2}$
$= 8x^5$
$= 8\left(\frac{1}{2}\right)^5$
$= 0.25$
$= \frac{1}{4}$

16. a. $10^3 \cdot 10^4 = 10^{3+4} = 10^7$ eggs

b. First find the ratio of surviving eggs to total eggs.

$\frac{10^6}{10^7} = 10^{6-7} = 10^{-1}$

Multiply this ratio by 100, or 10^2, to find the percent of surviving eggs.

$10^{-1} \cdot 10^2 = 10^{-1+2} = 10^1$

10% of eggs survive.

17. Let y represent the amount of money in the account and t represent the number of years since the $75 was deposited.

$y = 75(1 + 0.03)^t = 75(1.03)t$

When $t = 3$,

$y = 75(1.03)^3 \approx 81.9545$

After 3 years, the investment will be worth $81.95.

18. a. Initial amount: 54

Decay rate: 0.05

Decay factor: $1 - 0.05 = 0.95$

b. *Sample answer:* $y = 54(0.95)^t$

t	0	1	2	3	4
y	54	51.3	48.7	46.3	44

There were 44 members 4 years after 2000, or in 2004.

19. a. $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(1.569 \times 10^6)^3$

$= \frac{4}{3}\pi(1.569)^3(10^6)^3$

$\approx 16.179 \times 10^{18}$

$= 1.6179 \times 10^1 \times 10^{18}$

$\approx 1.618 \times 10^{19}$

The volume of Europa is about 1.618×10^{19} m^3.

b. $d = \frac{m}{V}$

$= \frac{4.8 \times 10^{22}}{1.618 \times 10^{19}}$

$= \frac{4.8}{1.618} \times \frac{10^{22}}{10^{19}}$

$\approx 2.967 \times 10^3$

The average density d of Europa is about 2.967×10^3 kg/m^3.

c. *Sample answer:* The order of magnitude of the mass of Europa is 10^{22} and the order of magnitude of the volume of Europa is 10^{19}.

$d = \frac{m}{V} = \frac{10^{22}}{10^{19}} = 10^{22-19} = 10^3$

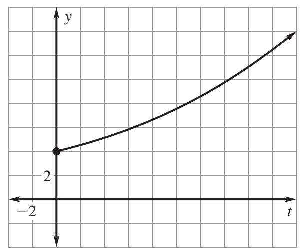
This approximation would underestimate the density calculated in part (b).

20. a. Let y represent the number of lily pads and t represent the number of days since the start.

$y = 4(1 + 0.065)^t = 4(1.065)^t$

b. $y = 4(1.065)^t$

t	0	5	10	15	20
y	4	5.5	7.5	10.3	14.1



Domain: $0 \le t \le 20$

Range: $4 \le y \le 14.1$

c. Using the trace feature on the graphing calculator you will find that $y = 10$ when $t \approx 15$. So, there will be 10 lily pads on about day 15.